(A)微積分:

1. A function $f: \mathbb{R} \to \mathbb{R}$ and $f(x) = x^3 + ax^2 + bx + c$,

(a) If at $x=1$, f has a maximum and at $x=3$, f has a minimum, then what are the values of a and b? (5%)

(b) If at $x=4$, f has a minimum and at $x=1$, f has a point of inflection, then what are the values of a and b? (5%)

2. Find the following Indefinite Integrals:

(a) $\int \frac{x+2}{(1-x)^4}\,dx$ (5%)

(b) $\int \sin\frac{x}{2}\cos\frac{3}{2}x\,dx$ (5%)

3. Evaluate

(a) $\left|\int_0^4 (x-1)\,dx\right| = ?$ (5%)

(b) $\int_0^4 |x-1|\,dx = ?$ (5%)

(B) 線性代數

4. (6%) Prove the following equality.

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$$

5. (7%) Solve the simultaneous linear equations:

$$x_1 + x_2 + 5x_3 + 3x_4 + x_5 = 5$$
$$2x_1 + x_2 + 7x_3 + 4x_4 + x_5 = 7$$
$$x_1 - x_2 - x_3 - x_4 - x_5 = 5$$
$$3x_1 + 2x_2 + 12x_3 + 7x_4 + 2x_5 = 12$$

6. (12%) Let $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$. We wish to find the eigenvalues of A and their associated eigenvectors.

7. (10%) Let W be the subspace of R^4 with orthonormal basis $\{w_1, w_2, w_3\}$, where

$w_1 = \left(\frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}\right)$, $w_2 = (0, 0, 1, 0)$ and $w_3 = \left(\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right)$

a) Write the vector $\mathbf{v} = (1, 0, 2, 3)$ as $\mathbf{w} + \mathbf{u}$, with $\mathbf{w}$ in W and $\mathbf{u}$ in $W^{\perp}$ (orthogonal complement of W).

b) Let $\mathbf{v} = (1, 2, -1, 0)$. Find the distance from $\mathbf{v}$ to W.

(C) 機率與統計

8. (35 %) Let the continuous random variable X follows a distribution D. Two types of questions often take place.

(a) Find the value of a such that $P(X \geq a) = \alpha$ for a given value of α, and

(b) Find the value of α such that $P(X \geq a) = \alpha$ for a given value of a.

Let Y follow $N(\mu, \sigma^2)$ and U follow the exponential distribution with parameter λ. Answer the following questions.

(i) (25%) To solve for both types of $P(U \geq a) = \alpha$ equations you need only a calculator. However, you need the standard normal distribution table available in almost every Probability and Statistics textbook to solve both types of $P(Y \geq a) = \alpha$ equation. Explain why?

(ii) (5%) Give the step-by-step procedure that applies the standard normal distribution table to solve the value of a such that $P(Y \geq a) = \alpha$ for a given value of α.

(iii) (5%) Give the step-by-step procedure that applies the standard normal distribution table to solve the value of α such that $P(Y \geq a) = \alpha$ for a given value of a.