機率與統計

1. (35 %) Let the continuous random variable X follows a distribution D. Two types of questions often take place.
   (a) Find the value of a such that $P(X \geq a) = \alpha$ for a given value of α, and
   (b) Find the value of α such that $P(X \geq a) = \alpha$ for a given value of a.

   Let Y follow $N(\mu, \sigma^2)$ and U follow the exponential distribution with parameter λ. Answer the following questions.

   (i) (25%) To solve for both types of $P(U \geq a) = \alpha$ equations you need only a calculator. However, you need the standard normal distribution table available in almost every Probability and Statistics textbook to solve both types of $P(Y \geq a) = \alpha$ equation. Explain why?

   (ii) (5%) Give the step-by-step procedure that applies the standard normal distribution table to solve the value of a such that $P(Y \geq a) = \alpha$ for a given value of α.

   (iii) (5%) Give the step-by-step procedure that applies the standard normal distribution table to solve the value of α such that $P(Y \geq a) = \alpha$ for a given value of a.

2. (15 %) The Central Limit Theorem follows. Let $Y = \sum_{i=1}^{n} X_i$, where X_1, X_2, ..., X_n are s-independent (statistically independent) random variables with means μ_i and variances σ_i^2. Let $\mu_Y = \sum_{i=1}^{n} \mu_i$ and $\sigma_Y^2 = \sum_{i=1}^{n} \sigma_i^2 < \infty$. Let $U = \dfrac{Y - v_Y}{\sigma_Y^2}$. Then, U asymptotically follows the standard normal distribution regardless the distributions of X_1, X_2, ..., X_n.

   Please try to apply the Central Limit Theorem to explain the phenomena that random errors observed in the laboratory are often normally distributed.

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3. (6%) Express the following LP model in standard form.

Maximize $Z = 2x_1 + 3x_2 + 5x_3$

Subject to

$$x_1 + x_2 - x_3 = -5$$

$$-6x_1 + 7x_2 - 9x_3 \geq 4$$

$$x_1 + x_2 + 4x_3 \leq 10$$

and $\quad x_1$ unrestricted, $x_2 \geq 0$, $x_3 \geq 0$

4. (8%) Consider the following LP:

Maximize $Z = x_1 + 5x_2 + 3x_3$

Subject to

$$x_1 + 2x_2 + x_3 = 3$$

$$2x_1 - x_2 = 4$$

and $\quad x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

(a) Write the associated dual problem.

(b) Given the information that the optimal basic variables are x_1 and x_3, determine the associated optimal dual solution.

5. (10%) For a transportation problem as follows.

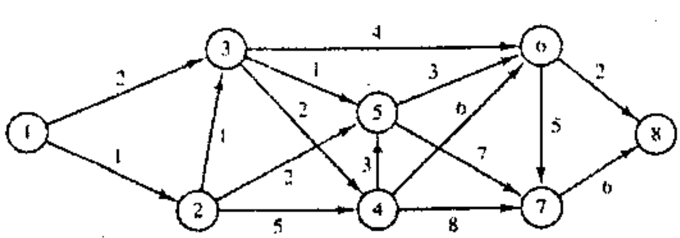
			Supply
0	2	1	6
2	1	5	7
2	4	3	7
5	5	10	

Demand

(a) Compare the starting solutions obtained by the northwest-corner, least-cost, and Vogel methods.

(b) Solve the problem starting with the northwest-corner solution.

6. (10%) The following network gives the distances in miles between pairs of cities 1, 2, ... and 8. Find the shortest route between cities 4 and 8.



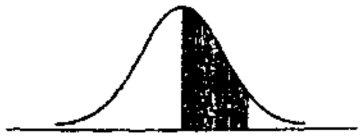
7. (10%) Perform a PERT analysis of the project given below and find the probability of finishing the project between 17 and 22 days inclusive: (Hint: You may refer to the normal curve area table at the end of the problem part.)

Activity	Predecessors	Duration Estimates (days) (a, m, b)
1	—	1, 4, 5
2	—	2, 3, 4
3	1	6, 10, 13
4	1	6, 6, 7
5	2	2, 2, 2
6	3	1, 2, 3
7	4, 5	5, 8, 9
8	2	12, 16, 19

8. (6%) The manager of a new fast-food restaurant wants to quantify the arrival process of customers by estimating the fraction of interarrival time intervals that will be (a) less than 2 minutes, (b) more than 3 minutes, and (c) between 2 and 3 minutes. Arrivals in similar restaurants occur at the rate of 35 customers per hour. The interarrival time exponentially distributed.

(背面仍有題目,請繼續作答)

TABLE 4 Normal Curve Areas



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Source: Abridged from Table I of A. Hald. *Statistical Tables and Formulas* (New York: John Wiley & Sons, Inc.), 1952. Reproduced by permission of A. Hald and the publisher.