一. (20%)

Find the general solution of the following differential equation :

(a) $y'' + 5y' = xe^{-x} \sin(3x)$

(b) $y'' + 6y' + 9y = -\cos(x)$; $y(0) = 1$, $y'(0) = -6$.

二. (15%)

Solve the following system by using "Laplace Transform method." $x_1'' = \frac{5}{2}x_2 - \frac{13}{2}x_1 + 2[1 - H(t-5)]$

$\qquad x_2'' = -\frac{13}{2}x_2 + \frac{5}{2}x_1$

with $x_1(0) = x_2(0) = x_1'(0) = x_2'(0) = 0$

$\qquad$ H is the unit step function.

三. (10%)

For $\varphi(x, y, z) = 18xyz + e^x$, Compute

(a) $\nabla \varphi = ?$

(b) $\nabla \times (\nabla \phi) = ?$

四. (15%)

Two lines L_1 and L_2 are given respectively by parametric equations :

$$x = 1 + 6t, \quad y = 2 - 4t, \quad z = -1 + 3t$$

and
$$x = 4 - 3p, \quad y = 2p, \quad z = 4p - 5$$

the parameters t and p vary over all real values

Please calculate the angle between these two lines.

五. (10%)

Find the equation of the tangent plane and normal line to the surface $x^2 - y^2 + z^2 = 0$ at the point $(1, 1, 0)$.

六. Calculate the following determinants :   (10%)

$$\begin{vmatrix} 5 & 3 & 0 & 0 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 9 & 1 & 0 & 0 \\ 0 & 0 & 2 & 2 & -4 & 0 & 0 \\ 0 & 0 & 1 & -8 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{vmatrix}$$

七. (20%)

Solve the boundary value problem

$$\frac{\partial^2 y}{\partial t^2} = 3\frac{\partial^2 y}{\partial x^2} + 2X \quad (0 < x < 2, \ t > 0)$$

$$y(0,t) = y(2,t) = 0 \quad (t > 0)$$

$$y(x,0) = \frac{\partial y}{\partial t}(x,0) = 0 \quad (0 < x < 2)$$

Hint : Set $Y(x.t) = y(x,t) + h(x)$