Calculus (2004)

Note: No partial credits will be given without evidence revealing that correct methods are applied.

1. (a) Let

$$f(x) = \begin{cases} x+1, x < 2 \\ x+3, x > 2 \end{cases}.$$

   Give $\lim_{x \to 2} f(x)$. Explain your answer. (10 points)

   (b) Let $f(x) = \dfrac{\sin(t)}{t}$. Give $\lim_{t \to 0} f(x)$. Explain your answer. (10 points)

2. Let $f(x) = \sin^{-1}(x) \cdot \cos(x)$. Find df/dx. (20 points)

3. (a) Give $\int \dfrac{e^x}{x} dx$. (10 points)

   (b) Give $\int \tan^3(x) \sec^4(x) dx$. (10 points)

4. Let $9x^2 - 18x + 4y^2 + 16y = 11$. Is it an equation of an ellipse, parabola, or hyperbola? (10 points)

5. Let $\bar{a}$, $\bar{b}$, and $\bar{c}$ be 3×1 vectors. Let $\bar{b} = (1,0,0)$ and $\bar{c} = (0,1,0)$. Solve the equation $\bar{a} \cdot (\bar{b} \times \bar{c}) = 0$ for $\bar{a}$. Is the solution for $\bar{a}$ unique? Explain your answer. (30 points)