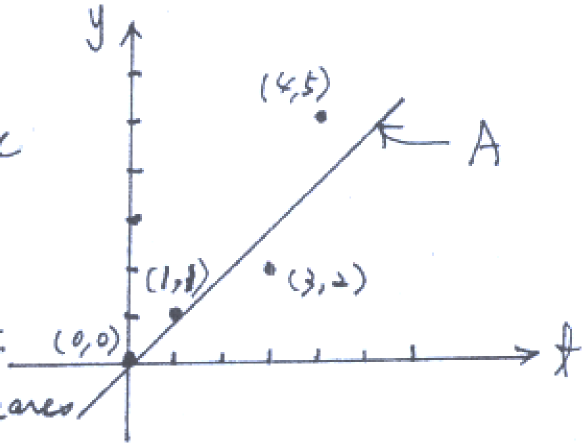
線性代數

1. For $A = \begin{bmatrix} 2 & 0 & 0 & 3 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 5 & 1 & 1 & 9 \end{bmatrix}$, find det A.
[10%]

2. For $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$, find its eigenvalues and eigenvectors.
(10%)

3. Suppose we are given the four measurements marked on the figure:
[10%]
$y=0$ at $t=0$, $y=1$ at $t=1$,
$y=2$ at $t=3$, $y=5$ at $t=4$.
Please find a least-squares fitting line A for these four data. (Hint: The line A may be $y = a + bt$, you need to find a and b.)



(背面仍有題目,請繼續作答)

4. Solve the following simultaneous equations:

[10%]

$$\begin{cases} x_1 + 2x_2 - 3x_3 + x_4 = 0 \\ 2x_1 + 5x_2 = 8 \\ x_1 - 14x_3 + 8x_4 = -15 \\ -2x_1 - 3x_2 + 14x_3 + 2x_4 = 10 \end{cases}$$

5. Prove that the vectors

[10%]

$$\begin{bmatrix} 1 \\ 3 \\ -7 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix}$$

span the same subspace of C^3 as do

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix}.$$

Probability (2004)

6. (20 %) Let Y follow $N(\mu,\sigma^2)$ and U follows the exponential distribution with parameter λ. Answer the following questions.

(i) (10%) To find the probability $\Pr(U \geq a)$ for a given value a, we need only a calculator. Why?

(ii) (10%) We cannot use a non-programmable calculator to find the probability $\Pr(Y \geq a)$ for a given value a. Explain why not?

7. (30 %) Let random variable U_i follow the exponential distribution with parameter $\lambda_i = i$ and all U_i's are mutually statistically independent, where $i = 1, 2$. Answer the following questions.

(i) (15%) Let $U^{\min} = \min\{U_1, 2U_2\}$. How are we going to find the distribution of $U^{\min}$? Give the distribution of $U^{\min}$ if you can.

(ii) (15%) Let $U^{\max} = Max\{U_1, 2U_2\}$. How are we going to find the distribution of $U^{\max}$? Give the distribution of $U^{\max}$ if you can.