

1. (15%) Find the general solution of $ydx + (3x - xy + 2)dy = 0$.
2. (15%) Solve the ordinary differential equation: $x^2y'' - 3xy' + 3y = \ln x$.
3. (15%) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ is the position vector of some space point and $r \equiv |\vec{r}|$, consider a differentiable scalar function $\phi(f)$, where f is function of r only. Evaluate $\nabla\phi[f(r)]$.

4. (15%) Find the right and left inverse for the matrix

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

5. (15%) Using the Laplace transform to find the solution of

$$\frac{d^2y}{dt^2} + 2t\frac{dy}{dt} - 4y = 1, \quad y(0) = 0, \quad \frac{dy}{dt}(0) = 0$$

6. (25%) Consider a slightly damped vibrating string that satisfies

$$\rho_0 \frac{\partial^2 u}{\partial t^2} = T_0 \frac{\partial^2 u}{\partial x^2} - \beta \frac{\partial u}{\partial t}$$

(a) What is the sign of β ? Why?

(b) Determine the solution (by separation of variables) which satisfies the boundary conditions

$$u(0, t) = 0 \quad \text{and} \quad u(L, t) = 0$$

and the initial conditions

$$u(x, 0) = f(x) \quad \text{and} \quad \frac{\partial u}{\partial t}(x, 0) = g(x).$$

Assume that the friction coefficient β is relatively small ($\beta^2 < 4\pi^2 \rho_0 T_0 / L^2$).