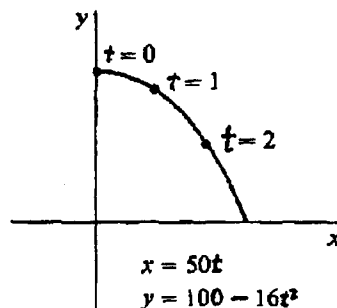


Calculus (2005)

1. (30 pts) The Euler formula: $e^{i\theta} = \cos \theta + i \sin \theta$. While this formula can be proven with a Maclaurin expansion, it is also applied to find $\cos(n\theta)$ and $\sin(n\theta)$ as polynomial functions of $\cos \theta$ and $\sin \theta$, where n is an integer.
- (a) (10 pts) Apply the above Euler formula to obtain $\cos(4\theta)$ and $\sin(4\theta)$ in terms of $\cos \theta$ and $\sin \theta$.
- (b) (10 pts) Give $\cos(k\theta)$ in terms of $\cos \theta$ and $\sin \theta$, where k is a positive integer.
- (c) (10 pts) Apply the Euler formula to derive $d \cos \theta / d \theta$ and $d \sin \theta / d \theta$.
Note: no credit will be given unless the Euler formula is applied in the derivation.
2. (15 pts) Solve the following problems.
- (a) (5 pts) Let $y = 1/\ln x$. Derive dy/dx .
- (b) (10 pts) Let $y = \int_3^{x^2+x} \frac{1}{t^3+1} dt$. Derive dy/dx .
3. (15 pts) A ball thrown horizontally from a 100 ft cliff at a velocity of 50 ft/sec follows the parametric equations
 $x = 50t$, and $y = 100 - 16t^2$, where x and y are in feet, and t in seconds.
 Find the slope of its path at time t .

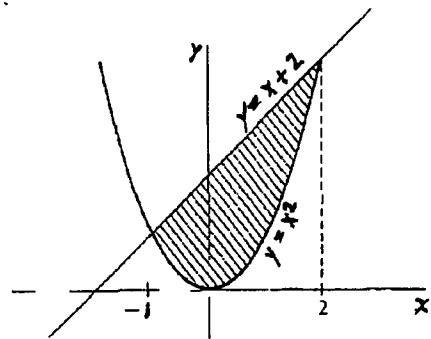
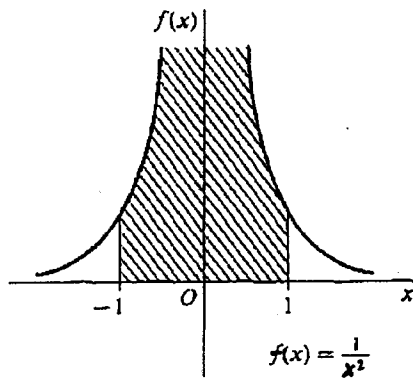


(背面仍有題目, 請繼續作答)

編號：G 347 系所：製造工程研究所丙組

科目：微積分

4. (20 pts) Solve the following problems.

(a) (10 pts) Let $B = \int_{-1}^1 \frac{1}{x^2} dx$. Give B .(b) (10 pts) Let A be the area of the region bounded above by $y = x + 2$ and below by $y = x^2$. Find A .

5. (20 pts) Answer the following questions.

(a) (5 pts) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{c}\right)^n = ?$, where $c > 0$.(b) (5 pts) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^c = ?$, where $c > 0$.(c) (10 pts) Is it true that $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = 1$? Explain your answer.