1. 求下列各函數之導數：

(a) $y = \ln|x^2 - 5x + 4|$. (5%)

(b) $y = (1 + \frac{1}{x})^{1/3}$. (5%)

2. 求下列方程式於指定點之切線方程式：

(a) $xy = 4$ 於點 $(-2, -2)$. (5%)

3. 試求下列各極限：

(a) $\lim_{x \to \pi} \frac{\sin x}{x - \pi}$ (5%)

(b) $\lim_{x \to 0} \frac{\sin x - x}{x^3}$ (5%)

4. 試求下列各不定積分：

(a) $\int \frac{x + \sqrt{x+1}}{\sqrt[3]{x+1}} dx$ (5%)

(b) $\int e^{\cos x} \sin x \, dx$ (5%)

5. 試求 $\int_1^3 t\sqrt{t^2 - 1} \, dt = ?$ (5%)

6. 試求曲線 $y = \sin x$ 與 x 軸在 $[0, 2\pi]$ 所圍區域之面積。(5%)

7. 試求由 y 軸，$y = 1$，$y = 4$，和曲線 $y = x^2$ 所圍區域繞 y 軸迴轉所得立體 S 之體積。(5%)

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8. (20%) Let $X \sim$ binomial (n, p), that is,

$$P(X=x)=\binom{n}{x}p^x(1-p)^{n-x},\ x=0,1,\ldots,n.$$

Find Var X.

9. Let X be a random variable with cdf F_X. The *moment generating function* (mgf) of X, denoted by $M_X(t)$, is

$$M_X(t)=Ee^{tX},$$

show that

(i) (15%) $EX^n = M_X^{(n)}(0)$, where we define $M_X^{(n)}(0)=\dfrac{d^n}{dt^n}M_X(t)\big|_{t=0}$.

(ii) (15%) $f(x)$ is a special case of the gamma pdf, where $\Gamma(\alpha)$ denotes the gamma function. Show that $EX=\alpha\beta$.

$$f(x)=\frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta},\ 0<x<\infty,\ \alpha>0,\ \beta>0$$