Physics (2005)

1. (20 pts) Electric dipole: a pair of charges lie in the x-y plane. The charge $+q$ is at coordinate x = 0, y = b; the charge $-q$ is at coordinate x = 0, y = − b as shown in Fig. 1 below.

   (a) (10 pts) Let $\vec{E}_x(r)$ be the electric field at the point of coordinate x = r, y = 0 on the x axis. Evaluate the electric field $\vec{E}_x(r)$ (magnitude and direction) on the x axis.

   (b) (3 pts) Show that for $|r| >> b$, $|\vec{E}_x(r)| \propto 1/r^3$, where $|\vec{E}_x(r)|$ is the magnitude of $\vec{E}_x(r)$.

   (2 pts) What is the direction in this limit? Note that r approaches positive and negative infinity.

   (c) (5 pts) Let $\vec{E}_y(s)$ be the electric field at the point of coordinate x = 0, y = s on the x axis. Evaluate the electric field $\vec{E}_y(s)$ (magnitude and direction) on the y axis.

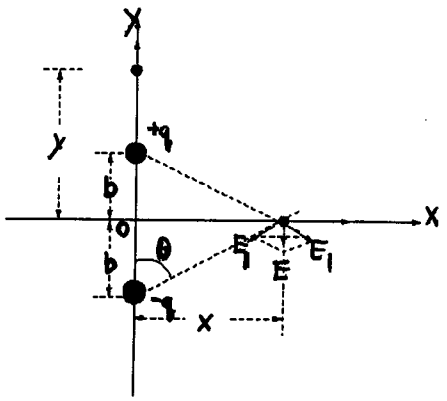


Figure 1. Electric field of a dipole.

2. (20 pts) A rod of length l_1 with line charge density λ_1 and a rod of length l_2 with line charge density λ_2 lie on the x axis. Their ends are separated by a distance D as shown in Fig. 2 below.

   (a) (10 pts) What is the force $\vec{F}$ between these charges?

   (b) (10 pts) Show that for $D >> l_1$ and $D >> l_2$, this force reduces to the Coulomb forces between a pair of point charges, $q_1 = l_1\lambda_1$, $q_2 = l_2\lambda_2$.

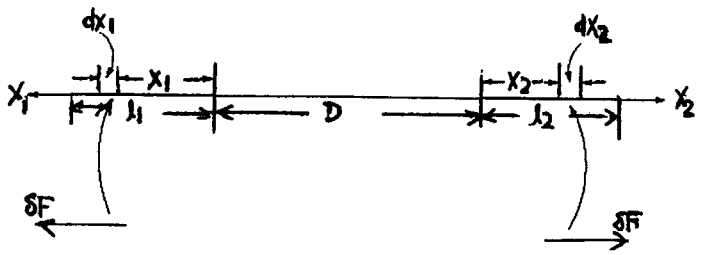


Figure 2. Electric field of a pair of line charges.

(背面仍有題目,請繼續作答)

3. (20 pts) Figure 3(a) below shows a cube resistor between two nodes that represent diagonally opposite corners of the cube, where resistors on all sides are the same (R0). The amount of currents on each side is indicated by two types of arrows (single arrow for I_1, and double arrows for I_2). Figure 3(b) shows its flattened circuit network.

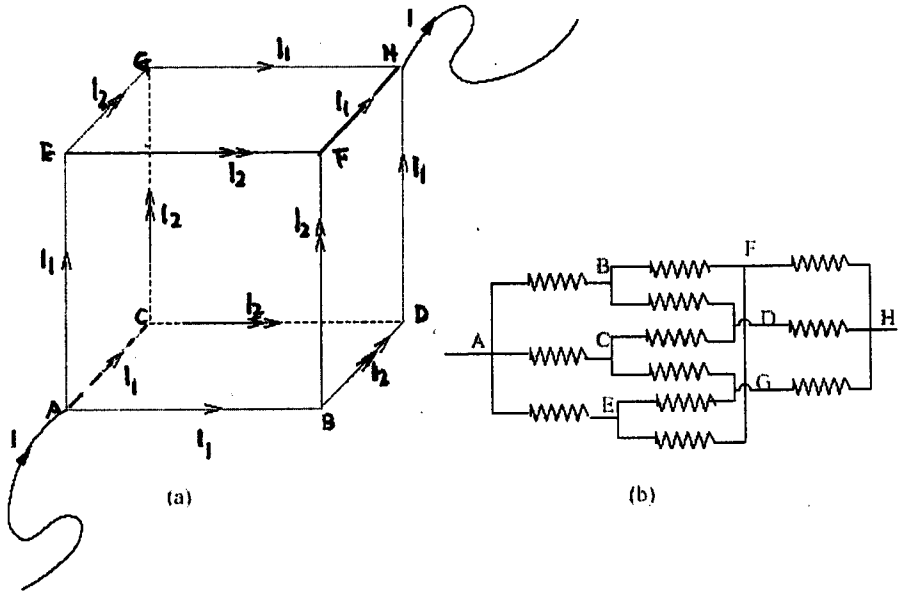
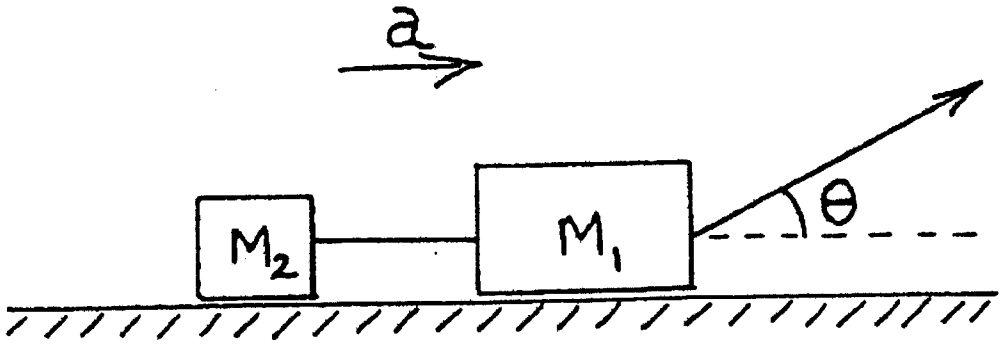


Figure 3. (a) Cube resistor between two nodes that represent diagonally opposite corners of the cube; (b) the flattened circuit network.

4. (20 pts) Let the masses of two blocks be M_1 and M_2, respectively. Let the coefficient between the blocks and blocks be κ. A force $\bar{F}$ pulls along the direction as indicated by the arrow resulting in an acceleration of $\bar{a}$. Give the magnitude of $\bar{F}$ and the tension of the rope that connects blocks.



5. (20 pts) Let ρ_1 be the mass density of a cubic solid block S_1 with a side length of ℓ. Let ρ_2 and ρ_3 be the mass densities of liquids L_2 and L_3, where $\rho_3 > \rho_1 > \rho_2$. In equilibrium, let $\ell_{S1\,in\,L2}$ be the length of ℓ immersed in liquid L_2. Express $\ell_{S1\,in\,L2}$ in terms of ρ_1, ρ_2, and ρ_3. (Ignore surface tension effects.)