

本試題是否可以使用計算機:  可使用,  不可使用 (請命題老師勾選)1. Solve for  $y(t)$  from the simultaneous equations

$$[20\%] \quad \begin{cases} y' + 2y + 6 \int_0^t z \, dt = -2u(t) \\ y' + z' + z = 0 \end{cases}$$

if  $y_0 = -5$  and  $z_0 = 6$ .

2. Find the solution of the equation  $(4D^2 + 16D + 17)y = 0$  for which  $y = 1$  when  $t = 0$  and  $y = 0$  when  $t = \pi$ .3. Show that the vectors  
 $[20\%] \quad v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix}$   
 are linearly dependent.4.  $X$  is a uniformly distributed random variable with mean 1 and variance 2.  $Y$  is a Gaussian random variable with mean 0 and variance 4. The correlation coefficient between  $X$  and  $Y$  is  $\rho = 1/2$ .Define:  $W = X + \alpha Y$  and  $V = X - \alpha Y$ .Find  $\alpha$  so that  $V$  and  $W$  are uncorrelated.5. Let  $y = \ln \sqrt{\frac{x^2+1}{x^2-1}}$ , find  $\frac{dy}{dx}$ .  
 $[5\%]$ 6. Let  $x^y = y^x$ , find  $\frac{dy}{dx}$ .  
 $[5\%]$ 7. Find  $\int \sqrt{1 - \cos x} \, dx$ .  
 $[5\%]$ 8. Find  $\int (x^e + e^x) \, dx$ .  
 $[5\%]$