

本試題是否可以使用計算機: 可使用, 不可使用 (請命題老師勾選)

Calculus (2006)

1. (30 pts) Let $\vec{a} = (1, 0, 0)$, $\vec{b} = (0, 0, 1)$, and \vec{c} is a 3×1 vector. Assume that \vec{a} , \vec{b} and \vec{c} has the following relationship: $\vec{c} \cdot (\vec{a} \times \vec{b}) = 0$. Give the solution set of \vec{c} . (Note: The solution set may contain one or more than one solutions of \vec{c} .)
2. (30 pts) The Euler formula: $e^{i\theta} = \cos \theta + i \sin \theta$.
 - (a) (10 pts) Apply the Euler formula to obtain $\cos(\theta/2)$ in terms of $\cos \theta$ and $\sin \theta$.
 - (b) (10 pts) Apply the Euler formula to obtain $\sin(3\theta)$ in terms of $\cos \theta$ and $\sin \theta$.
 - (c) (10 pts) Apply the Euler formula to derive $d \cos \theta / d\theta$ and $d \sin \theta / d\theta$.
Note: no credit will be given unless the Euler formula is applied in the derivation.
3. (20 pts) By definition, $df(\theta)/d\theta = \lim_{\Delta\theta \rightarrow 0} \frac{f(\theta + \Delta\theta) - f(\theta)}{\Delta\theta}$ if the limit exists.
Apply the above definition and the following formula to derive $d \sin \theta / d\theta$.
$$\sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta).$$
4. (20 pts) Solve the following problems.
 - (a) (10 pts) $\int \frac{\ln x}{x^2} dx$.
 - (b) (10 pts) $\int \frac{1}{x^{n+0.5}} dx$, where n is a non-negative integer.