本試題是否可以使用計算機： ☑可使用 ， □不可使用 （請命題老師勾選）

微積分

1. (10%) Obtain the *Jacobian* matrix for the following mapping: $w = x^2 y z$

2. (10%) Let $z = x^3 - 3x^2 y$, where x and y are functions of t such that for $t=5$, $x=7$, $y=2$, $dx/dt=3$, and $dy/dt=-1$. Find dz/dt for $t=3$.

3. (10%) If $u=f(x, y)$ and $x=r\cos\theta$, $y=r\sin\theta$, then show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

4. (15%) Let R be the quarter-circle, $0 \le y \le \sqrt{(1-x^2)}$, $0 \le x \le 1$, and let $f(x, y) = x^2 + y^2$. Solve $\int_R \int (x^2 + y^2)\,dx\,dy$

機率與統計

5. (20%) A closet contains n pairs of shoes. If $2r$ shoes are chosen at random ($2r<n$), what is the probability that:

  (i) there will be no complete pair in the sample, and

  (ii) there will be exactly one complete pair in the sample?

6. (20%) Proof

  (i) Let X have a geometric distribution. Show that $P[X>m+n|X>m] = P[X>n]$ for all nonnegative integers m, n.

  (ii) Let Let X have an exponential distribution. Show that $P[X>u+v|X>u] = P[X>v]$ for all positive numbers u, v.

7. (15%) 一個治療古柯鹼癮的三種治療方法的雙向表如下：

	成功	失敗	總數
去體敏	14	10	24
鋰鹽	6	18	24
安慰劑	4	20	24
總數	24	48	72

H_0：在所有古柯鹼成癮患者的母體當中，處理和戒癮成功之間並沒有關聯

請算出"卡方統計量（chi-square statistic）"（必要時請註明你的假設)