

1. (20 pts). Let $\vec{a} = (1,1,0)$, $\vec{b} = (1,0,1)$ be 3×1 vectors in the Euclidean space. In addition, let \vec{c} be a 3×1 vector in the Euclidean space. Assume that \vec{a} , \vec{b} and \vec{c} has the following relationship: $\vec{c} \cdot (\vec{a} \times \vec{b}) = 0$. Give the solution set of \vec{c} . (Note: The solution set may contain one or more than one solutions of \vec{c} .)
2. (20 pts) The Euler formula: $e^{i\theta} = \cos \theta + i \sin \theta$.
- (a) (10 pts) Apply the Euler formula to derive the following equality:
- $$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$
- (b) (10 pts) Apply the Euler formula to derive $d \cos \theta / d\theta$ and $d \sin \theta / d\theta$.
Note: no credit will be given unless the Euler formula is applied in the derivation.
3. (20 pts) Find the following areas.
- (b) (10 pts) Let A be the area of the region bounded above by $y = x + 2$ and below by $y = x^2$. Find A .
- (a) (10 pts) Let $B = \int_{-1}^1 \frac{1}{x^2} dx$. Give B .
4. (20 pts) Solve the following problems.
- (a) (10 pts) Let $y = 1/\ln x$. Give dy/dx .
- (b) (10 pts) Let $y = \int_3^{x^2+x} \frac{1}{t^3+1} dt$. Derive dy/dx
5. (20 pts) Evaluate the following limits:
- (a) (5 pts) $\lim_{n \rightarrow \infty} (1 + \frac{1}{c})^n = ?$, where $c > 0$.
- (b) (5 pts) $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^c = ?$, where $c > 0$.
- (c) (10 pts) Is it true that $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = 1$? Explain your answer.