1. Find the general solution of the following
[10%] differential equation :

$$y'' + 4y = 8x^2$$

2.
[10%] Solve for $y'' - 4y' + 4y = 0$ with $y(0) = 3$, $y'(0) = 4$.

3. Use Laplace transform to solve the following
[10%] linear system :

$$x'' - 2x' + 3y' + 2y = 4$$
$$2y' - x' + 3y = 0$$

with $x(0) = x'(0) = y(0) = y'(0) = 0$

4. Compute the determinant of
[10%] the following matrix :

$$A = \begin{bmatrix} 5 & 3 & 0 & 0 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 8 & 1 & 0 & 0 \\ 0 & 0 & 2 & 2 & -4 & 0 & 0 \\ 0 & 0 & 1 & -8 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

5.
[10%] Solve the system $\begin{cases} x_1 + 2x_2 + 3x_3 = 4 \\ 2x_1 + 5x_2 + 3x_3 = 5 \\ x_1 + 8x_3 = 9 \end{cases}$

via the Cramer's rule.

（背面仍有題目，請繼續作答）

6. [10%] Let $v_1 = \begin{bmatrix} 4 \\ 6 \\ 7 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, and let $u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, and $u_3 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$.

If $x = 2v_1 + 3v_2 - 4v_3$, determine the coordinates of x with respect to $\{u_1, u_2, u_3\}$.

7. [20%] Consider the matrix $Q = \begin{bmatrix} 3 & t & -1 \\ t & 4 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

(a) Calculate $\det(Q)$

(b) Find all possible $t \in \mathbb{R}$ for which Q is invertible.

(c) Find $t \in \mathbb{R}$ such that $(1, 0, 1)^T$ is an eigenvector of Q associated to the eigenvalue 2.

8. [20%] Given the measurements y_1, y_2, y_3, y_4 at distinct points t_1, t_2, t_3, t_4, respectively:

$y_1 = 0$ at $t_1 = 0$; $y_2 = 1$ at $t_2 = 1$;
$y_3 = 2$ at $t_3 = 3$; $y_4 = 5$ at $t_4 = 4$

find the straight line $y = a + bt$ which minimizes the sum of the squares of the errors. {Straight-Line Approximation with Least Squares Fitting of Data.}