1. (20%) A research firm tests the miles-per-gallon characteristics of three brands of gasoline. Because of different gasoline performance characteristics in different brands of automobiles, five brands of automobiles are selected and treated as blocks in the experiment; that is, each brand of automobile is tested with each type of gasoline. The results of the experiment (in miles per gallon) follow.

		Gasoline Brands		
		I	II	III
Automobiles	A	18	21	20
	B	24	26	27
	C	30	29	34
	D	22	25	24
	E	20	23	24

   a. At $\alpha = .05$, is there a significant difference in the mean miles-per-gallon characteristics of the three brands of gasoline? (Hint: from F table, 2 degrees of freedom numerator and 8 denominator, $F_{0.05} = 4.46$)

   b. Analyze the experimental data using the ANOVA procedure for completely randomized designs. Compare your findings with those obtained in part (a). What is the ad- vantage of attempting to remove the block effect? (Hint: from F table, 2 degrees of freedom numerator and 12 denominator, $F_{0.05} = 3.89$)

2. (14%) Data on the marital status of men and women aged between 20 and 29 were obtained as part of a national survey. The results from a sample of 209 men and 204 women are as follows. Use $\alpha= .01$ and test for independence between marital status and gender. What is your conclusion? (Hint: from χ^2 table, 2 degrees of freedom, $\chi^2_{.01} = 9.21$)

Gender	Marital Status		
	Single	Married	Divorced
Men	177	28	4
Women	141	57	6

3. (21%) Let the continuous random vector (X,Y) have joint density function (pdf)

$$f(x,y) = \begin{cases} 6xy^2 & 0 < x < 1 \text{ and } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

   (i) Compute the integral of $f(x,y)$ over the whole plane.

   (ii) Find the probability such that $P(X+Y \geq 1)$.

   (iii) Find the marginal pdf of X. What is the probability $P\left(\frac{1}{2} < X < \frac{3}{4}\right)$?

4. (15%) Let the continuous random vector (X,Y) have joint density function (pdf) $f(x,y) = e^{-y}$, $0<x<y<\infty$, calculate $P(X+Y \geq 1)$.

5. (15%) An employer is about to hire one new employee from a group of N candidates, whose future potential can be rated on a scale from 1 to N. The employer proceeds according to the following rule: a) Each candidate is seen in succession (in random order) and a decision is made whether to hire the candidate. b) Having rejected $m-1$ candidates ($m>1$), the employer can hire the mth candidate only if the mth candidate is better than the previous $m-1$. Suppose a candidate is hired on the ith trial. What is the probability that the best candidate was hired?

6. (15%) In a town of $n+1$ inhabitants, a person tells a rumor to a second person, who in turn repeats it to a third person, etc. At each step the recipient of the rumor is chosen at random from the n people. Find the probability that the rumor will be told exactly t times: i) before returning to the originator. ii) without repeated to any person.