編號： 240-1　　國立成功大學九十七學年度碩士班招生考試試題　　共 1 頁，第 1 頁

系所：製造工程研究所甲組　　科目：工程數學

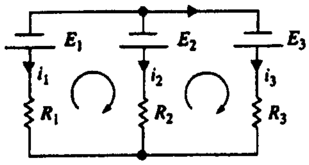
本試題是否可以使用計算機： ☑可使用 ， □不可使用 （請命題老師勾選）　　考試日期：0301，節次：3

Problem 1 (20 points)

Find the general solutions of the following equations:

(a) $y''' - 5y'' + 6y' = 8 + 2\sin x$;　　(b) $x^2y'' - xy' + y = x^3$.

Problem 2 (15 points)



(a) Show that the system of equations for the currents i_1, i_2, and i_3 in the circuit shown above is

$$\begin{aligned} i_1 + i_2 + i_3 &= 0, \\ -R_1 i_1 + R_2 i_2 &= E_2 - E_1, \\ -R_2 i_2 + R_3 i_3 &= E_3 - E_2, \end{aligned}$$

where R_k and E_k, $k = 1, 2, 3$, are constants.

(b) Express the system as a matrix equation $\mathbf{AX} = \mathbf{B}$, where $\mathbf{X} = (i_1, i_2, i_3)^T$.

(c) Show that the coefficient matrix $\mathbf{A}$ is nonsingular, and use $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$ to solve for the currents.

Problem 3 (15 points)

The temperature at a point (x, y) on a rectangular metal plate is given by $T(x, y) = 100 - 2x^2 - y^2$. Find the path a heat-seeking particle will take, starting at $(3, 4)$, as it moves in the direction in which the temperature increases most rapidly.

Problem 4 (20 points)

Use Laplace transform to solve

(a) the initial-value problem: $y'' - y' = e^t \cos t$, $y(0) = 0$, $y'(0) = 0$; and

(b) the integrodifferential equation $f(t) + 2\int_0^t f(\tau)\cos(t - \tau)\, d\tau = 4e^{-t} + \sin t$.

Problem 5 (20 points)

Poisson's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -h,$$

where the constant $h > 0$, occurs in many problems involving electric potential. Solve the above equation subject to the conditions

$$\begin{aligned} &u(0, y) = 0, \quad u(\pi, y) = 1 \qquad (y > 0); \\ &u(x, 0) = 0 \qquad (0 < x < \pi). \end{aligned}$$

Problem 6 (10 points)

Evaluate the contour integral $\oint_C \mathrm{Re}(z)\, dz$, where C is the circle $|z| = 1$ (counterclockwise).