※ 考生請注意：本試題 ☑可 ☐不可 使用計算機

Calculus (2008)

1. (20 pts) Do the following integrations.

   (a) (10 pts) $\int_0^1 \frac{e^x}{1+e^x}dx$

   (b) (10 pts) $\int \frac{1}{x\ln x}dx$

2. (20 pts) Answer the following questions.

   (a) (10 pts) $\lim_{x\to\infty}\frac{x}{\ln x}$

   (b) (10 pts) $\lim_{t\to 0^+}(\sin t)^t$

3. (20 pts) Give a numerical example that demonstrates the following property of vector product:

   Let $\vec{y}=\alpha\cdot\vec{a}+\beta\cdot\vec{b}$, where α and β are real numbers. Then, $\vec{y}\cdot(\vec{a}\times\vec{b})=0$.

4. (20 pts) Find the area of the region between the graphs of $f(x)=3x^3-x^2-10x$ and $g(x)=-x^2+2x$, where $x\in[-2,\ 2]$. It is noted that $f(x)-g(x)=0$ while $x=-2,\ 0,\ 2$.

5. (20 pts) Solve the differential equation $dy/dx=2x/y$.