

系所組別： 製造資訊與系統研究所乙組

考試科目： 工程數學(乙組)

考試日期： 0307，節次： 3

※ 考生請注意：本試題 可 不可 使用計算機

1. Find the general solution of  $y'' + y' - 2y = e^x$ .  
[10%]

2. Solve  $\begin{cases} (D-5)x - 4y = -5t^2 + 6t + 25, & x(0) = 0 \\ (D-2)y - x = -t^2 + 2t + 4, & y(0) = 0 \end{cases}$   
[20%] where  $D = \frac{d}{dt}$ .

3. [10%] If  $f(t) = \begin{cases} 0, & t < 0 \\ 2t, & 0 \leq t < 3 \\ t^2, & 3 \leq t \end{cases}$ , find the Laplace Transform of the given function  $f(t)$ .

4. [20%] By applying the Laplace Transformation technique to the following differential equation:  
 $ty'' + (4t - 2)y' - 4y = 0, y(0) = 1, y'(0) = -2,$   
derive its solution.

Also reply that whether there exist a unique solution or not.

5. [10%] Find the largest possible number of independent vectors from the following vector set.

$$v_1 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \quad v_5 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \quad v_6 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

(背面仍有題目,請繼續作答)

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6. Let  $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$

[20%]

- Find all the eigenvalues of  $A$ .
- Find all the eigenvectors of  $A$  associated with each eigenvalue of  $A$ .
- Find the invertible matrix  $P$  such that  $P^{-1}AP$  is diagonal.
- Compute  $A^{100}$ .

7. Given  $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ , find

[10%] complex scalars  $s$ , such that  $A + sB$  is not invertible.