系所組別： 製造資訊與系統研究所甲組

考試科目： 工程數學

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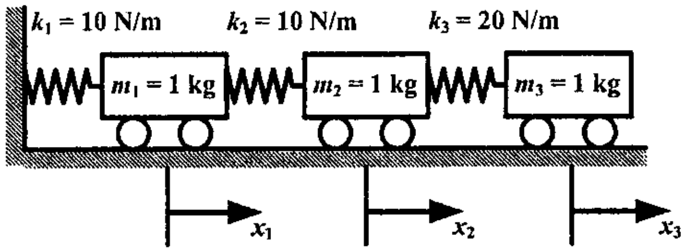
Problem 1 (20 points)



A three-mass system with $m_1 = m_2 = m_3 = 1$ kg is connected by three springs having spring constants $k_1 = k_2 = 10$ N/m and $k_3 = 20$ N/m, as shown above. Set up the differential equations, and determine the natural frequencies (eigenvalues) and mode shapes (eigenvectors) of this system.

Problem 2 (15 points)

Solve the following initial-value problem:

$$xy'' + y' = x, \quad y(1) = 1, \quad y'(1) = -1/2.$$

Problem 3 (15 points)

Use Laplace transform to solve the following system of equations:

$$\begin{cases} x'' + y'' = e^{2t}, \\ 2x' + y'' = -e^{-2t}, \end{cases}$$

subject to the initial conditions $x(0) = x'(0) = y(0) = y'(0) = 0$.

Problem 4 (10 points)

Suppose that m is the mass of a moving particle. Newton's second law of motion can be written in vector form as

$$\mathbf{F} = m\mathbf{a} = \frac{d}{dt}(m\mathbf{v}) = \frac{d\mathbf{p}}{dt},$$

where $\mathbf{F}$ is the force acting upon the particle, $\mathbf{v}$ and $\mathbf{a}$ are its velocity and acceleration, respectively, and $\mathbf{p} = m\mathbf{v}$ is its *linear momentum.*

(a) Show that the *torque* $\mathbf{T} = \mathbf{r} \times \mathbf{F}$, with $\mathbf{r}$ being the position vector of the particle, is the time rate of change of the *angular momentum* $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ of the particle with respect to the origin.

(b) Explain why the angular momentum of a particle under the action of a *central force*—i.e., a force directed along the position vector of the particle—is conserved.

Problem 5 (20 points)

Verify Green's theorem in the plane for $\oint_C \left\{(3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy\right\}$, where C is the boundary of the region defined by $x = 0$, $y = 0$, and $x + y = 1$.

(背面仍有題目,請繼續作答)

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Problem 6 (20 points)

The temperature in a rod of unit length, with heat transfer from its right boundary into a surrounding medium kept at a constant (zero) temperature, is determined from

$$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad (0 < x < 1,\ t > 0),$$

$$u(0,t) = 0, \quad \left.\frac{\partial u}{\partial x}\right|_{x=1} = -hu(1,t) \quad (t > 0),$$

$$u(x,0) = 1 \quad (0 < x < 1),$$

where $h > 0$ is a constant heat transfer coefficient. Solve for $u(x,t)$.