系所組別 製造資訊與系統研究所甲組
考試科目 工程數學

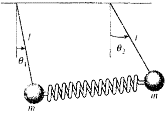

※ 考生請注意：本試題 ☑可 □不可 使用計算機

**Problem 1** (20 points)

Solve the following initial-value problem:

$$\begin{cases} y'' + 4y' + 4y = (3+x)e^{-2x}, \\ y(0) = 2, \quad y'(0) = 5. \end{cases}$$

**Problem 2** (20 points)




Suppose that two identical pendulums, having length $l$ and point mass $m$, are coupled by a *linear* spring with force constant $k$, as sown in the above schematic.

(a) Show that, when the displacement angles $\theta_1(t)$ and $\theta_2(t)$ are small, the system of linear differential equations describing the motion is

$$\ddot{\theta}_1 + \omega^2\theta_1 = K(\theta_2 - \theta_1), \qquad \ddot{\theta}_2 + \omega^2\theta_2 = K(\theta_1 - \theta_2),$$

where $\dot{(\cdot)} = d(\cdot)/dt$, $\omega^2 = g/l$, and $K = k/m$, with $g$ being the gravitational acceleration.

(b) Use *Laplace transform* to solve the system with the following initial conditions:

$$\theta_1(0) = \theta_0, \quad \dot{\theta}_1(0) = 0, \quad \theta_2(0) = \psi_0, \quad \dot{\theta}_2(0) = 0,$$

where $\theta_0$ and $\psi_0$ are constants.

**Problem 3** (20 points)

Use *Stokes' theorem* to evaluate

$$\oint_C \left( z^2 e^{x^2} dx + xy^2 dy + \tan^{-1} y \, dz \right),$$

where $C$ is the circle $x^2 + y^2 = 9$ running *counterclockwise* on the $x$-$y$ plane.

**Problem 4** (20 points)

Use *diagonalization* to solve the following system:

$$\frac{d\mathbf{X}}{dt} = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix} \mathbf{X} + \begin{pmatrix} e^t \\ e^t \end{pmatrix}$$

where $\mathbf{X} = (x_1(t) \;\; x_2(t))^T$

(背面仍有題目,請繼續作答)

※ 考生請注意：本試題 ☑可 □不可 使用計算機

**Problem 5** (20 points)

Use *Fourier transform* to solve the heat equation

$$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \qquad (-\infty < x < \infty, t > 0),$$

subject to the initial condition $u(x,0) = e^{-x^2}$ $(-\infty < x < \infty)$. **Hint:** Use the result

$$\int_{-\infty}^{\infty} e^{-x^2/4p^2} e^{i\alpha x}\, dx = 2\sqrt{\pi}\, p\, e^{-p^2\alpha^2}$$