系所組別： 機械工程學系甲、乙、丙、丁、戊組

考試科目： 工程數學　　　　　　　　　　　　　　　考試日期：0219，節次：3

※ 考生請注意：本試題 ☑可　□不可　使用計算機

1. Please solve the following two ODEs: (25%):

(a) $xy' + y = 3x^3 y^3$; $y(2) = \sqrt{\dfrac{\pi}{2}}$

(b) $x^2 y'' - 5xy' + 8y = 2\ln x$; $y(1) = 1$, $y'(1) = 1$

2. For the given matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(a) Find the eigenvalues and eigenvectors of A. (15%)

(b) Find A^4 and the inverse matrix of A. (10%)

3. What is the definition of the derivative of f(z), when z = x + i y is a complex number? Proper explanation is needed to get the full score. (6%)

If z = x + i y, and f(z) = u(x, y) + i v(x, y), what is the sufficient condition for f(z) to be differentiable at z? (6%)

4. What is the definition of sin z, when z = x + i y is a complex number? (5%)

If f(z) = sin z, what is the derivative of f(z)? (3%)

You need to show the procedure from definition to the answer. (5%)

(背面仍有題目,請繼續作答)

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5. If the governing equation of a simply-supported beam can be described as follows:

$$EI \frac{d^4 y}{d x^4} + K y = g(x)$$

where E, I and K are all constants, $y(x)$ is the beam deflection in Y-direction, x is the axial position in X-direction, and $g(x)$ is the applied load which is specified by the figure below:
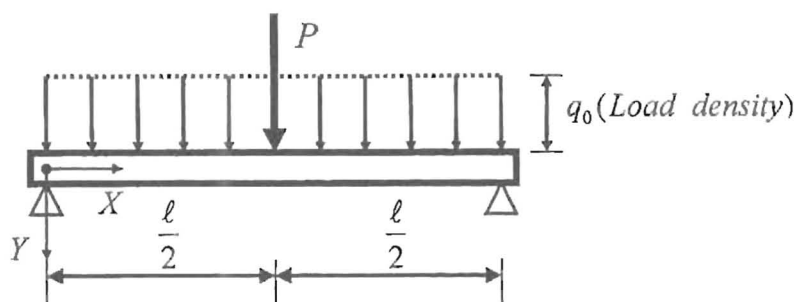
, find

(1). Fourier Sine Series of $g(x)$. It is noted that P and q_0 are both

constant. (9 %)

(2). $a_n = ?$, where a_n is the coefficient of the deflection

$$y(x) = \sum_{n=1}^{\infty} a_n \sin \frac{n\pi x}{\ell} .$$

(10 %)

(3). the first 3 terms of Fourier Sine Series of $y(x)$ if $x = \frac{\ell}{2}$.

(6%)