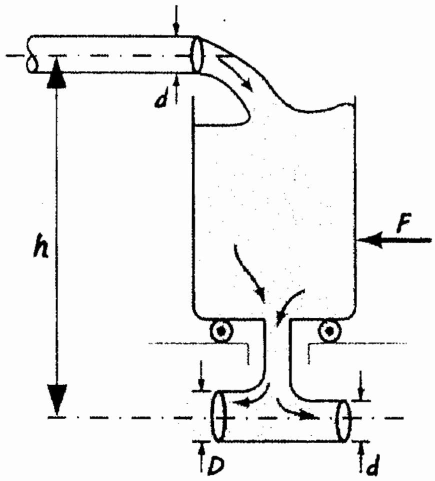
※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (25 pts)



Water flows steadily into and out of a tank that sits on frictionless wheels as shown above. Suppose that the inlet pipe is at a height h above the two discharge pipes, but is at a negligible height over the water surface inside the tank. (Assuming $D, d \ll h$, it is appropriate to neglect the distance between the centerlines of the left and right discharge pipes when $D \neq d$.) Let us also neglect viscous losses.

(a) Determine the volumetric flowrate from the left discharge pipe, and that from the right one.

(b) Calculate the horizontal force F required to keep the tank motionless.

(c) Also determine the diameter D of the left discharge pipe so that the tank will remain motionless when $F = 0$.

2. (25 pts) Consider laminar, steady flow of an incompressible fluid past an infinite flat plate (placed at $y = 0$). The fluid, however, is withdrawn by a steady constant suction of velocity V through the plate, which is slightly porous. (So the y-component of the fluid velocity $v = -V$ at $y = 0$.) In this case the boundary layer does not grow with distance along the plate (i.e., the x-axis) but remains constant, so that $\partial u/\partial x = 0$, with u being the x-component of the fluid velocity.

(a) The boundary layer thickness, δ, is expected to depend on the fluid density ρ and viscosity μ, and the suction velocity V. Use dimensional analysis to determine such functional dependence.

(b) Starting with the boundary layer equations, show that

$$u(y) = U\left[1 - \exp\left(-\frac{Vy}{\nu}\right)\right] \quad (y \geq 0),$$

where U is the free-stream velocity and $\nu = \mu/\rho$ is the kinematic viscosity.

(c) Determine the displacement and momentum thicknesses.

(d) Calculate the drag coefficient on one side of the plate.

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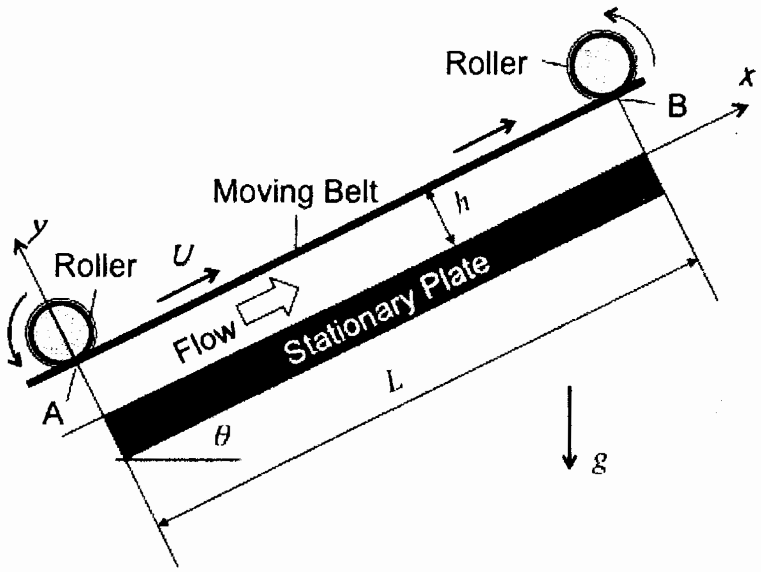
3. (35 pts) Chemical fluid is driven upwards along an inclined plane by a belt moving at constant velocity, U. The width of the belt, b, is much smaller than the length, L. The bottom plate is stationary and inclined at an angle θ with respect to horizontal plane. The flow of the fluid between points A and B in the figure below is assumed to be laminar and fully developed. Density and kinematic viscosity of the fluid are ρ and ν, respectively. The height of the gap between the belt and the bottom plate is h. Specific gravity, g, is pointing vertically downwards. The absolute pressure of the surroundings is P_{atm}. Directions of x and y axes are shown in the figure.



(a) Using Navier Stokes equations to derive expressions for pressure field and velocity profile. Do it step by step showing every assumption and simplification.

(b) What is the power used by the belt to draw up the fluid along the inclined plate between point A and B?

(c) Where will there be deviations from fully developed assumption? Briefly assess how good is the fully developed assumption in this flow?

(d) If the wall shear stress τ_w is a function of ρ, ν, $g\sin\theta$, h, U, using Buckingham Π theorem to find the dimensionless groups that represent this problem, i.e. find dimensionless shear stress as a function of other dimensionless groups, $\Pi_1=f(\Pi_2, \Pi_3, \ldots)$. Then find the function "f".

4. (15 pts) Compare the behavior of fully developed laminar flow and fully developed turbulent flow in a horizontal pipe under the following conditions. Answer the following questions and provide brief explanations.

(a) For the same flow rate, which will have the larger centerline velocity? Why?

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(b) If the pipe discharges to atmosphere, what would you expect the trajectory of each discharge stream to look like? Sketch your expectation for each case.

(c) For the same flow rate, which flow would give the larger wall shear stress, τ_w? Why?

(d) Sketch the shear stress distribution τ/τ_w as a function of radius for each flow.

(e) For the same Reynolds number, which flow would have the larger pressure drop per unit length?