國立成功大學 104 學年度碩士班招生考試試題

系所組別：機械工程學系甲乙丙丁戊組

考試科目：工程數學

考試日期：0211，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. **(15%) If $A=\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$, and an orthogonal matrix P can diagonalize A to a diagonal matrix D such that $D = P^{\mathrm{T}}AP$, find P and D as well as calculate A^{100}.**

2. **Solve the following ODEs:**

   (10%) (2.1) $y'' + 4y' + 4y = 2e^{-2x}$

   (10%) (2.2) $x^2 y'' + xy' - y = x^2 e^x$

3. (20%) Solve, by use of **Laplace transform**, the one-dimensional diffusion problem formulated below:

$$\begin{aligned}
&\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2} && (0 < x < \infty,\ t > 0); \\
&u = U \cdot H(t) && (x = 0), \\
&u \to 0 && (x \to \infty), \\
&u = 0 && (t = 0),
\end{aligned}$$

where $u = u(x,t)$, ν and U are constant, and $H(t)$ is the Heaviside unit-step function.

Hint: For $f(t) = \operatorname{erfc}\left(\dfrac{a}{2\sqrt{t}}\right)$, with $\operatorname{erfc}(\eta) = 1 - \dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^{\eta} e^{-\xi^2}\, d\xi$ being the complementary error function, the Laplace transform $F(s) = \displaystyle\int_0^{\infty} e^{-st} f(t)\, dt = \dfrac{\exp(-a\sqrt{s})}{s}$.

4. (15%) The inverse Laplace transform can be written as

$$f(t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{st}F(s)\,ds,$$

where the path of integral with respect to s is a vertical line parallel to the imaginary axis, and is on the right of all the singularities of $F(s)$, in the complex s plane.

Now, by use of the **residue theorem**, calculate the inverse Laplace transform of $\dfrac{s^3}{s^4-a^4}$.

5. **(15%)** Determine the solution T(x, t) of the following boundary value problem.

$$\frac{\partial^2 T}{\partial t^2}=\frac{\partial^2 T}{\partial x^2}+T, \quad 0<x<\pi \quad \text{and} \quad t>0$$

with the boundary conditions

$$T(0,t)=T(\pi,t)=0$$

and the initial conditions

$$T(x,0)=\begin{cases} x, & 0<x<\pi/2 \\ \pi-x, & \pi/2\le x<\pi \end{cases}, \qquad \frac{\partial T}{\partial t}(x,0)=0$$

6. **(15%)** Determine the solution T(r, θ) of the following boundary value problem.

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}=0, \quad 0<\theta<\pi \quad \text{and} \quad 0<r<1$$

with the boundary conditions

$$T(r,0)=T(r,\pi)=0, \quad 0<r<1$$

and

$$T(1,\theta)=1, \quad 0<\theta<\pi$$