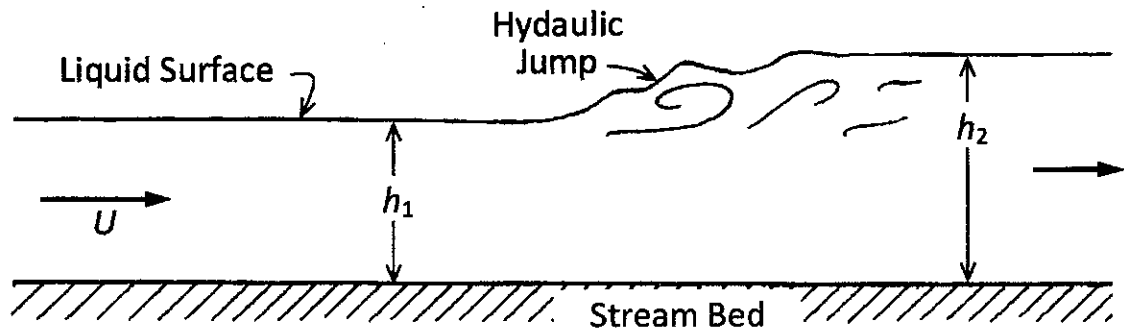


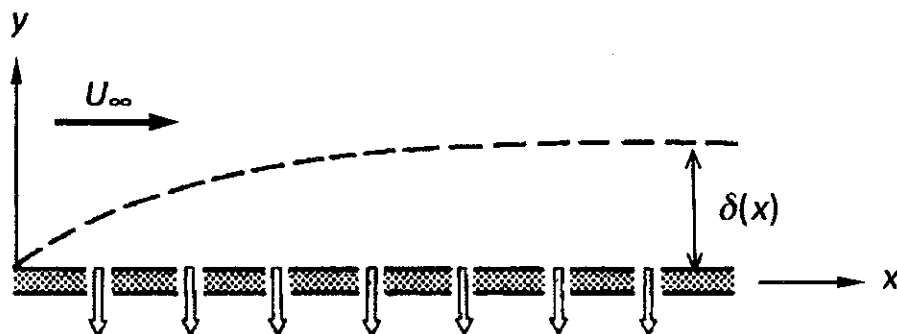
※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (25%) A hydraulic jump is a sudden increase in the depth of a liquid stream (which in this case is flowing over a horizontal stream bed with atmospheric pressure air everywhere above the liquid):



The depth increases suddenly from h_1 to h_2 downstream of the jump. The jump itself is often turbulent and involves viscous losses so that the total pressure downstream is less than that of the upstream flow. Find the ratio of the depth, h_2/h_1 , in terms of the upstream velocity, U , the depth, h_1 , and the gravitational acceleration, g . Assume the flows upstream and downstream have uniform velocity parallel to the stream bed and that the shear stress between the liquid and the stream bed is zero. The liquid is incompressible. What condition on the value of U^2/gh_1 must hold for a hydraulic jump like this to occur?

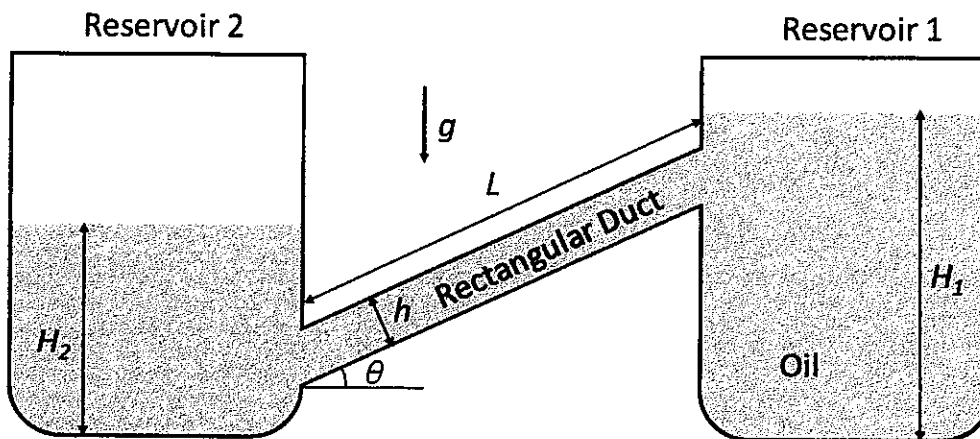
2. (25%) A laminar boundary layer forms on a porous flat surface that removes fluid from the main flow at a constant velocity, V , as shown.



Using the approximate integral method and assuming similarity of the velocity profile, i.e. that $u/U_\infty = f(y/\delta)$ where f is not a function of x , find a relation between the frictional coefficient, $c_f = 2\tau_w/\rho U_\infty^2$, and the quantities V , U_∞ , $d\delta/dx$, and α where α is the profile parameter

$$\alpha = \int_0^1 (f - f^2) d\left(\frac{y}{\delta}\right)$$

3. (50%) Oil is flowing between two very large reservoirs through a duct as shown in the figure below. Length of the rectangular duct connecting the two reservoirs is L . Height of the duct cross-section is h , and the width (in the direction into the paper) is w ; while w is much larger than h . The angle between the pipe and the horizontal axis is θ . Density and dynamic viscosity of the oil are ρ and μ , respectively. Specific gravity, g , is pointing vertically downwards in the figure. Assume flow in the duct is laminar, and becomes fully developed after a very short entrance region. Friction factor for the flow in the duct is given as $f = 96/Re_h$, where Re_h is Reynolds number based on the hydraulic diameter of the duct.



- (1) (15%) Find the oil flow rate, Q , through the duct in terms of the parameters given in the problem description.
- (2) (25%) Derive the expression of the velocity profile for flow in the fully developed section in the duct connecting the reservoirs. List all necessary assumptions and show the derivation step by step.
- (3) (10%) Sketch the instantaneous velocity profiles for fully developed laminar and turbulent flows in the duct on the same scale, and briefly explain their differences.