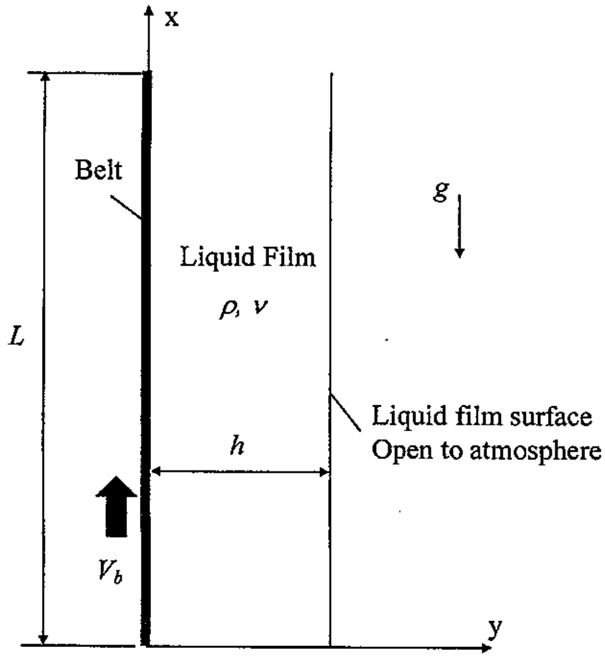
※考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (50%) In a coating process, a belt is moving with a constant velocity, V_b. The belt is drawing up a liquid film with a thickness h. The flow of the liquid is assumed to be steady, laminar and fully developed. The width of the belt, b, is in the z direction in the figure below, and is much larger than the film thickness. The liquid film is open to the atmosphere and the friction between the liquid and the air could be neglected. Density of the liquid is ρ, and the kinematic viscosity is ν. The gravity, g, is in the negative x direction in the figure. Length of the belt is L.

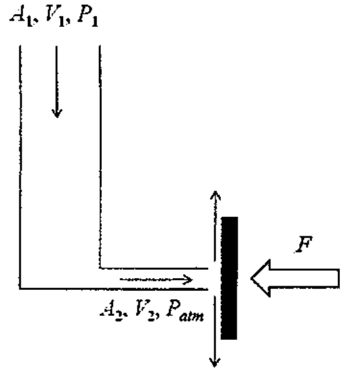


(1) Using Navier-Stokes equations to show that the pressure field within the liquid is constant. (10 %)
(2) Show that the velocity profile in the liquid film is $u = [g/(2\nu)]y^2 - (gh/\nu)y + V_b$. Do it step by step, show every assumption and simplification. (15 %)
(3) If the shear stress on the belt surface τ_w is a function of g, h, ρ, find the dimensionless groups that represent this problem using Buckingham Π theorem, i.e. find $\Pi_1 = f(\Pi_2, \Pi_3, \ldots)$ (10%)
(4) Using results in part (2) and (3), find the function f in part (3). (10%)
(5) What is the power used by the belt to draw up the liquid? (5%)

2. (30%) For a fluid flowing through a circular tube, show that the wall shear stress can be expressed as the following: $\tau_w = \frac{\Delta PR}{2L}$, where Δp is the pressure difference along the tube, L is the length of the tube, and R is the radius of the tube. (3%) Also, please make a plot showing the difference of the Newtonian and non-Newtonian fluids on the shear stress - shear rate curve. (3%) Please explain that under which conditions the non-Newtonian fluid can be treated as the Newtonian fluid. (4%). Now, if we can model

the behaviors of a non-Newtonian fluid using the power-law concept which is described as $\tau = -k\left(\frac{du}{dr}\right)^n$ where n and $k$are the constants. Please derive the corresponding flow rate in terms of the aforementioned variables and constants. (20%)

3. (15%) An air flow strikes a vertical plate at an outlet. Please determine the amount of force that is necessary to be applied to keep the plate in place. Assume the flow is steady, frictionless, and incompressible. The density of the air flow is 1.3 kg/m^3. Assume A_1(0.3 m^2) is three times larger than A_2, and $P_1 - P_{atm}$ = 2000 Pa. (10%) If you need to consider the effect of friction, how does it affect the flow profile? Please provide necessary mathematical formulations to support your statement. (5%)



4. (5%) Please provide details to set-up an experiment for measuring the viscosity of an unknown fluid.