※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15%) Solve the following ODE by <u>variable substitution</u>

$x^2 y'' - 5xy' + 8y = 2\ln x$　；　$y(1) = y'(1) = 0$

2. (15%) Solve the system of linear ODEs by <u>matrix diagonalization (similarity transform of eigen-matrix)</u>

$y_1' = 3y_1 + 3y_2 + 8$

$y_2' = \quad y_1 + 5y_2 + 4e^{3t}$

3. (15%) Use the Laplace transform to solve the following integrodifferential equation:

$$y'(t) = 1 - \sin t - \int_0^t y(\tau)d\tau, \quad y(0) = 0.$$

4. (20%)

(a) Evaluate

$$\oint_C \frac{1}{(z-1)^2(z-3)} dz$$

where the contour C is the circle $|z| = 2$.

(b) Show that the complex mapping $w = \sinh z$ (with $z = x + iy$) is conformal. Then find the image of the strip $-\pi/2 \le y \le \pi/2$, $x \ge 0$, under this mapping. What is the image of a vertical line segment in the strip?

5. Determine the solution of the following initial-boundary value problem: (20%)

$$\frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \quad \text{for} \quad 0 < x < 1, \, 0 < y < 1, \, t > 0,$$

$T(x, 0, t) = T(x, 1, t) = 0$ for $0 < x < 1, \, t > 0,$

$T(0, y, t) = T(1, y, t) = 0$ for $0 < y < 1, \, t > 0,$

and $T(x, y, 0) = f(x, y) = 10\sin(x)\cos(2y)$

6. Determine the solution $\bar{T}(x,s)$, in the Laplace transform domain ($\bar{T}(x,s)$, is the Laplace transform of $T(x, t)$) of the following initial-boundary value problem by Laplace transform technique: (15%)

$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \quad \text{for} \quad 0 < x < L, t > 0,$$

$$T(x, 0) = T_0 = \text{constant}, \quad \text{for } 0 < x < L,$$

$$T(L,t) = \frac{\partial T}{\partial x}(0,t) = 0 \quad \text{for} \quad t > 0.$$