國立成功大學
115學年度碩士班招生考試試題

編　號：50

系　所：機械工程學系

科　目：工程數學

日　期：0203

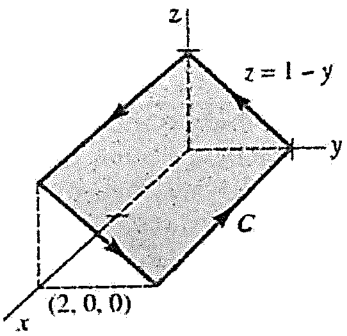
節　次：第 3 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. Solve $dy/dx = y \ln|x|$. (10%)

2. Solve $y'' + y = \cos x - \sin x$. (10%)

3. Evaluate $\oint_C F \cdot dr$, $F = z^2 y \cos xy\, \vec{i} + z^2 x(1 + \cos xy)\, \vec{j} + 2zy \sin xy\, \vec{k}$; Let C be the boundary of the planar region on the plane $z = 1 - y$, $0 \le x \le 2$, $0 \le y \le 1$, oriented according to the right-hand rule as shown below. (10%)



4. The Fibonacci sequence is defined by: (10%)

$$F_n = F_{n-1} + F_{n-2} \ (n \ge 2), and\ F_1 = 1,\ F_0 = 0$$

It can be written in matrix form as:

$$\begin{bmatrix} F_n \\ F_{n-1} \end{bmatrix} = A \begin{bmatrix} F_{n-1} \\ F_{n-2} \end{bmatrix}$$, where A is 2 x 2 matrix.

(1) Find the matrix A.

(2) Find the eigenvalues and eigenvectors of A.

Express the eigenvectors as the columns of a matrix in the form $\begin{bmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{bmatrix}$ (where $\lambda_1 > \lambda_2$)

5. Evaluate $\iint_R x e^{y^2}\, dA$, over the region R in the first quadrant bounded by the graphs of $y=x^2$, $y=4$, $x=0$. (10%)

6. Determine the Laplace transform F(s) of the function defined as follows. (10%)

$$f(t) = \begin{Bmatrix} 1\,;\ 0 \le t < 2 \\ -3\,;\ 2 \le t < 3 \\ t^2;\ t \ge 3 \end{Bmatrix}$$

7. To solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ $u(x,0) = 0, u_t(x,0) = 0, \ 0 < x < \infty, \ 0 < t < \infty$ and (10%)

$$u(0,t) = -\int_0^t u(0,s)ds + e^{-2t} - 1$$

8. Expand the periodic function $f(x) = \begin{Bmatrix} 1; & 0 < t < 1 \\ -1; & 1 < t < 2 \end{Bmatrix}$ with period 2 in a Fourier series in complex form.

(10%)

9. (a) Find the root of $1+z^4=0$ (5%)

(b) Assume we choose the principal branch of $\sin^{-1}z$ to be that one for which $\sin^{-1}0 = 0$, show that

$$\sin^{-1}z = \frac{1}{i}\ln(iz + \sqrt{1-z^2})$$ (5%)

10. To prove $\int_0^\infty \frac{\cos wx}{1+w^2}dw = \frac{\pi}{2}e^{-x} \ (x > 0)$ (10%)