(5%) 1. $\vec{F}$ is a Conservative Vector field. Choose the right statements from the following: (答錯一題倒扣一分)

(A) $\nabla \times \vec{F} = 0$

(B) $\int_C \vec{F} \cdot d\vec{r}$ is independent of the path C.

(C) $\oint_C \vec{F} \cdot d\vec{r} = 0$, where C is a close path.

(D) $\vec{F} = \nabla f$, where f is a scalar function.

(4%) 2. On interval $I \equiv (-\infty, \infty)$

$U_1(x) = x^2$, and, $U_2(x) = \begin{cases} -x^2, & x \le 0 \\ x^2, & x > 0 \end{cases}$

are functions $U_1(x)$ and $U_2(x)$ linearly dependent on each other over I ?

(4%) 3. Given the Matrix

$$H(x) = \begin{bmatrix} 1 & 2 & x-2 \\ x & 4 & 0 \\ 2 & x & -3 \end{bmatrix}$$

what is the rank of the matrix $H(x)$?

(5%) 4. A is a Matrix

$\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$, Choose the right statement from the following: (答錯一題倒扣一分)

(A) $A^2 - 2A + 3I = 0$,

(B) $A = 2I - 3A^{-1}$,

(C) $A^3 = A - 6I$,

(D) $A^4 = 4A^2 - 9A + 12I$,

where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(5%) 5. Choose the functions that have a Laplace Transform from the following: (答錯一題倒扣一分)

(A) $\operatorname{erf}(\sqrt{t})$, B) e^{2t}

(C) $t^{-1/2}$ D) $\int_0^t \frac{\sin x}{x} dx$,

(5%) 6. 試求 $\oint_C [\bar{z} - \operatorname{Re}(z)] dz$

在此 C: $|z| = 2$ $z = x + iy$.

(6%)13. (A) 何種複變函數必需滿足 Cauchy-Rieman Condition 並寫出 Cauchy-Rieman Condition 之關係式

(B). 如果此類函數滿足 Cauchy-Rieman Condition 應該亦能滿足那一類型之偏微分方程式, 試証明之。

(9%)14. 試解釋二階線性常微分方程式

$$\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0$$

① 受齊次邊界條件

② 受非齊次邊界條件

時. 在何種情況下有解, 又何種微分方程式及邊界條件具特徵值及特徵函數解.

(3%)15, 二階偏微分方程式

$$A\frac{\partial^2 u}{\partial x^2} + 2B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} = f\left(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right)$$

試由常係數 A、B、C 之條件來分類此二階偏微分方程式.

(10%) 7. Show that any equation

$$y'' + 2p(x)y' + q(x)y = 0$$

Can be reduced to the "Canonical form"

$$v'' + r(x)v = 0$$

by a change of dependent variable $y = a(x)v$;

that is, determine $a(x)$ and $r(x)$.

(9%) 8. 試求 $\int_0^\infty \frac{\cos(2x)}{x^4+1}\,dx$

(14%) 9. 試解.

$$\frac{\partial^2 y}{\partial t^2} = -\frac{\partial^4 y}{\partial x^4} + f(x) \qquad (0<x<\pi,\ t>0)$$

$$y(0,t) = y(\pi,t) = 0, \quad \frac{\partial^2 y}{\partial x^2}(0,t) = \frac{\partial^2 y}{\partial x^2}(\pi,t) = 0$$

$$y(x,0) = 0, \quad \frac{\partial y}{\partial t}(x,0) = 0$$

(5%) 10. 假設 C 爲圍繞原点之任意規則簡"閉曲綫(regular closed curve). 試求

$$\int_C \left(\frac{x}{x^2+y^2} + x^2\right)dx + \left(\frac{y}{x^2+y^2} - 2y\right)dy$$

(8%) 11. Solve the differential equation

$$\frac{d^2x}{dt^2} + 2Dw\frac{dx}{dt} + w^2x = \frac{P_0}{m}U(t)$$

Subject to Initial values $x(0) = 0$, $\left.\frac{dx}{dt}\right|_{t=0} = 0$

where, D, w, P_0, m, Constant, $U(t)$: unit step function.

12. Find Laplace Transform.

(3%) (A) $\mathcal{L}\left[\frac{1-e^{-t}}{t}\right] = ?$

(2%) (B) $\mathcal{L}\left[\frac{1-e^{-t}}{t^2}\right] = ?$

(3%) (C) $\mathcal{L}^{-1}\left[\ln\frac{s+a}{s+b}\right] = ?$