1. Answer as indicated:

   (a) Why can't stream functions be used for three-dimensional flows? (3%)

   (b) What is meant by circulation? How is vorticity related to circulation? (5%)

   (c) Relate vorticity to velocity and to shear stress. (3%)

   (d) How can the temperature of a fluid be increased without heat transfer? Give an example. (3%)

   (e) Explain an import use of dimensional analysis in fluid dynamics. (3%)

   (f) What is the shape factor of a boundary layer flow? What is the significance of the curve for shape factor versus Reynolds number? (5%)

2. Incompressible liquid flows steadily through a pipe of varying diameter, $D(x)$. The pipe contains a section of porous wall of length L, where liquid is supplied at the rate

$$q(x) = q_o \frac{x}{L}$$

with q_o expressed as volume flow rate per unit length. The liquid velocity in the pipe at the entrance to the porous section is U_o. The liquid supplied in the porous section has no axial component of velocity. The pressure is kept constant along the porous section. Evaluate the velocity distributions for flow through the porous section and obtain an expression for the required diameter variation $D(x)$. (20%)

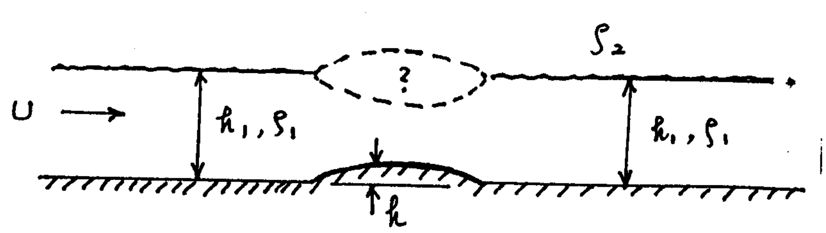
3. Let a turbulent boundary layer flow be from the leading edge to the trailing edge of a smooth flat plate. The shape of the velocity profile is similar at all positions along the plate. If the wall shear τ_w is inversely proportional to x^n, where n is a positive constant, show that the turbulent boundary layer thickness obeys the power law $\delta \propto x^{1-n}$. (8%)

4. Consider first a two layer system in two dimensions, with a layer of depth h_1, density ρ_1 flowing with a velocity U in the x-direction under a deep layer at rest. Suppose that an obstacle is introduced, with height $h=h(x)$ above the horizontal bottom, and length $L>>h$ so that the flow can be regarded as uniform at any section. Let Q be the flow rate and assume a constant

   (1) Find the relation between Q, h_1, h. (5%)

   (2) Plot the curve for $\frac{1}{2}\frac{Q^2}{\frac{g(\rho_1-\rho_2)}{\rho_1}h_1^2}$ as function of h_1. (3%)

   (3) Determine the shape of heavy layer flow over an obstacle when $F=(\frac{Q^2}{g'h_1^3})^{\frac{1}{2}}<1$ and >1 where $g'=\frac{g(\rho_1-\rho_2)}{\rho_1}$. (3%)



5. A flow in a channel of varying cross section is considered to be steady, one-dimensional and isentropic, gravitational forces too are negligible because we are dealing with a gaseous medium. The fluid density ρ, velocity V, cross-sectional area A and flow pressure P are variable properties along the flow direction, but the density, velocity and pressure are regarded as constants at any cross section.

(1) prove that the flow satisfies

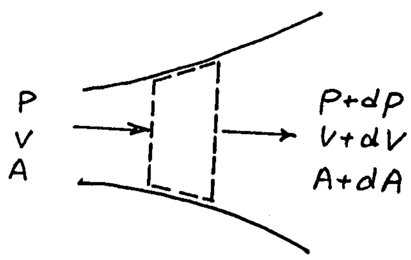
$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \qquad (3\%)$$

(2) Prove that the momentum equation in the direction of flow yields

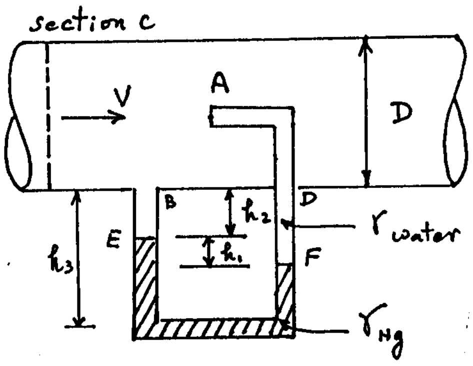
$$dP\,(1 - M^2) = \rho V^2 \frac{dA}{A} \qquad (5\%)$$

where M is Mach number.

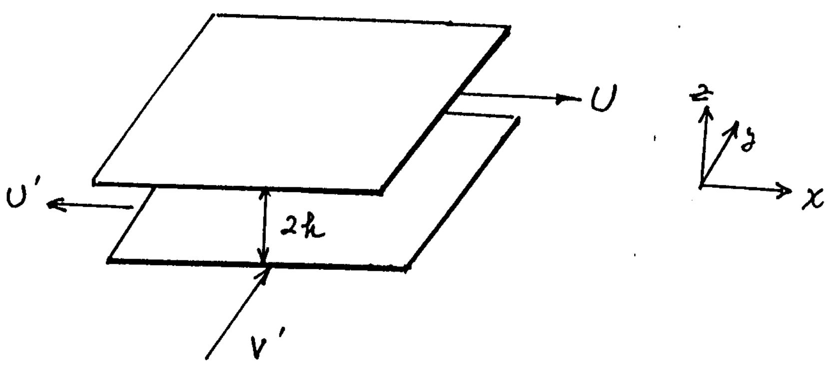
(3) indicate the pressure and velocity variations along the flow direction, all comments are made to subsonic and supersonic flows in diverging and converging channels. (5%)



6. Water flows in a horizontal pipe of constant diameter. Assume that the water is frictionless and that the velocity is constant across and along the pipe. A tube connected to the side of the pipe bends into a U shape and returns to the inside of the pipe. The right end of the tube bends with the open end of the tube facing upstream. The U part of the tube is filled partially with mercury. The part of the tube connected to a hole in the side of the pipe at point B measures the pressure of the liquid at that point. The velocity at B is undisturbed and equals the uniform velocity of the water. The other end of the tube is a stagnation point and so it measures the staguation pressure. The tube is filled with water except for the part filled with mercury. Compute the average velocity of the water. (γ: specific weight) (10%)



7. Two parallel flat plate are a distance 2h apart. The top plate is moving with a velocity U to the right and the lower plate is moving with a velocity U' to the left and V' in the positive y-direction. Calculate the location and value of the maximum velocity of the flow between the two plates. (7%)



8. Any arbitrary analytical function f of complex variable $z = x + iy$ has a simple complex representation

$$f = A Z^n$$

where A and n are constants, if we let the real part of f(z) be velocity potential $\psi(r, \theta)$ and the imaginary part be stream function $\psi(r, \theta)$, i.e

$$f(z) = \phi(r, \theta) + i\psi(r, \theta) \qquad 0 \le \theta \le 2\pi$$

(1) Determine the stream function $\psi(r, \theta)$ and velocity potential $\phi(r, \theta)$. (5%)

(2) Plot the streamline for $n = 3, 2, \frac{1}{2}$ and indicate the correct flow direction. (4%)