1. Find the general solution of the differential equation

$$y'' + \omega^2 y = r(t),$$

where

$$r(t) = \begin{cases} 1 & 0 < t < \pi, \\ 0 & \pi < t < 2\pi, \end{cases}$$

and

$$r(t+2\pi) = r(t). \qquad (20\%)$$

2. Evaluate

$$\int_0^{2\pi} \frac{d\theta}{\sqrt{2} - \cos\theta} \quad . \qquad (13\%)$$

3. For a given function $f(x) = \frac{3}{2} x + \frac{3}{2} \ell$, plot the corresponding representation of Fourier sine series, Fourier cosine series and complete Fourier series on the interval $[-\ell, \ell]$, respectively. (10%)

4. Considering a boundary value problem,

$$\frac{d^2}{dx^2} y(x) + \lambda\, y(x) = 0, \quad \lambda > 0, \; y(0) = 0 \text{ and}$$

$$y(1) + \frac{d}{dx} y(1) = 0.$$

(a) Determine its eigenvalues and the corresponding eigenfunctions.
(b) Show the orthogonality of the eigenfunctions by using properties of integrals. (10%)

5. In a temperature field, $T(x,y) = x^3 - 3xy^2$, heat flows in the direction of maximum decrease of temperature T. Find this direction at (2,1) position. (5%)

6. Determine the value of the surface integral $\iint_S \vec{F} \cdot \vec{N}\, d\sigma$,

where $\vec{F} = x^2 \vec{i} - (1+2x)\vec{j} + z\vec{k}$, S is the lateral surface of that portion of the cylinder $x^2 + y^2 = 1$ for which $0 \leq z \leq 1$. (8%)

7. Transform the quadratic form in x, y and z,

$$f(x,y,z) = 4xy + 4xz + 4yz,$$

to principal axes. (10%)

8. Solve the problem,

$$\frac{\partial Q(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} = 0, \quad 0 < x, \; 0 < t,$$

$$Q(0,t) = Q_o(t), \quad 0 < t,$$

$$Q(x,0) = 0 \quad , \quad 0 \leq x. \qquad (24\%)$$