

1. (20%) The following figure shows a child dangling on a swing. Derive a mathematical model of the system and explain from control point of view why the swing can be oscillated continuously.

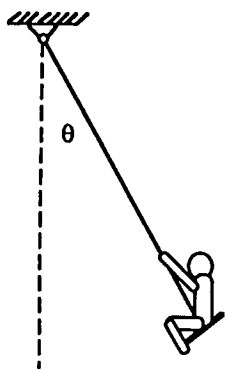


Figure 1. The model of a child on a swing

2. (20%) The transfer ^{function} of a plant is given by

$$G(s) = \frac{s+1}{(s-1)(s+\tau)}$$

where the parameter of the plant τ is changing between -0.5 and 0.5. Design a controller for this system such that (a) the closed-loop system is stable and (b) the steady-state error of the system for unit-step command is less than 3%.

3. (10%) Draw the closed-loop poles of the followed plant with unit feedback when the parameter τ is varied from 1 to 100.

$$G(s) = \frac{1}{(s+1)(s+\tau)}$$

4. A unity feedback system has the following open-loop frequency response. (20%)

ω	2	3	4	5	6	8	10
$ G(j\omega) $	7.5	4.8	3.15	2.25	1.70	1.00	0.64
$\angle G(j\omega)$	-118°	-130°	-140°	-150°	-157°	-170°	-180°

- Evaluate the gain margin and phase margin of the system.
- Determine the change in gain required so that the gain margin of the system is 20 db.
- Determine the change in gain required so that the phase margin of the system is 60 deg.

5. Estimate the transfer function for the Bode plot of magnitude shown in Fig.5. Assume that it is minimum phase. (20%)

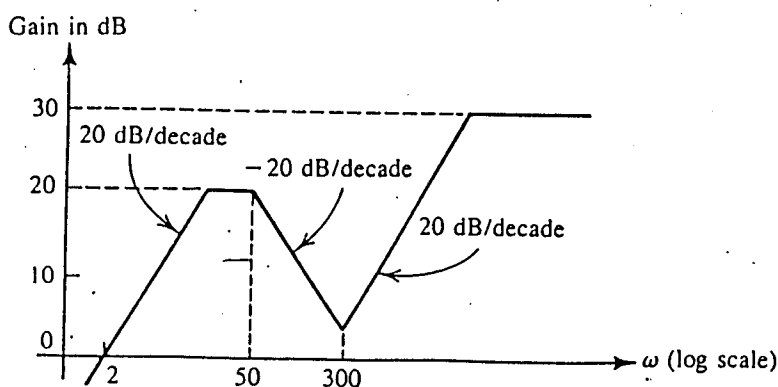


Fig.5 Bode Plot

- If an open loop transfer function of a feedback control system is $G(s)H(s)$, (10%)
 - there is a counterclockwise encirclement of the $-1+0j$ point at its Nyquist path, please use Nyquist criterion to explain: under which conditions is this system stable? and when is it unstable?
 - Can you explain the concept of relative stability briefly? How can you explain it with Bode-Plot of the system?