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1. A shear layer of unknown thickness grows along the sharp flat plate(Figure 1). The no-slip wall condition retards the flow, making it into a velocity profile u(y), which merges into the external velocity U = constant at a thickness y = $\delta(x)$. (a) Show that the drag force D(x) on the plate of width b is given by

$$D(x) = \rho b\int_{0}^{\delta} u(U-u)\,dy$$

(b) Define the momentum thickness θ and show that

$$\tau_w = \rho U^2\frac{d\theta}{dx}$$

(c) To get a numerical result for laminar flow, the velocity profile is assumed to have an approximately parabolic shape

$$u(x,y) \approx U\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right), \qquad 0\le y \le\delta(x)$$

Show that the boundary layer thickness $\delta(x)$ can be expressed as

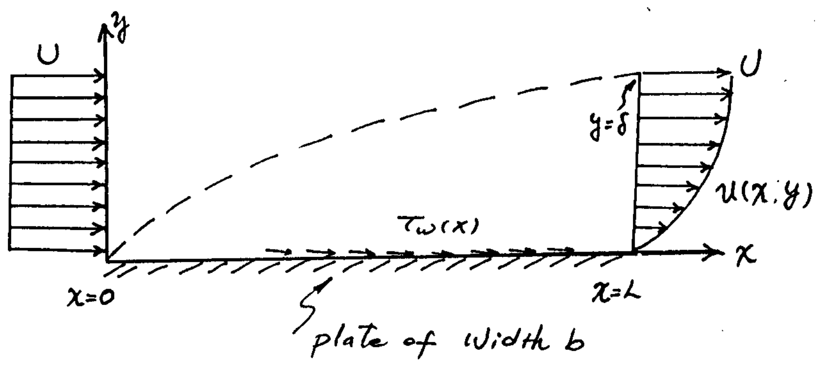
$$\frac{\delta}{x} \approx \frac{5.5}{Re_x^{1/2}}, \qquad Re_x= \frac{Ux}{\nu}$$

(d) Show that the skin-friction coefficient C_f can be written as

$$C_f \approx \frac{0.73}{Re_x^{1/2}}$$

(e) Describe the advantage and the disadvantage of using momentum integral theory to solve the boundary layer problem.(30%)

Figure 1.



2. Potential flow past a wedge of half-angle θ leads to an important application of laminar boundary layer theory called the Falkner-Skan flows. Let x denote distance along the wedge wall, as shown in Figure 2, and let $\theta = 10^{\circ}$. The pressure at point O is P_o. For a flow around a corner of arbitrary angle 2θ, the complex potential is

$$f(z) = A\cdot r^n[\cos(n\theta)+i\cdot\sin(n\theta)]$$

where A and n are constants.

(a) Using the above equation to find the variation of surface velocity U(x) along the wall.

(b) Is the pressure gradient adverse or favorable?

(c) Write the expression of pressure at any point x along the wall.(20%)

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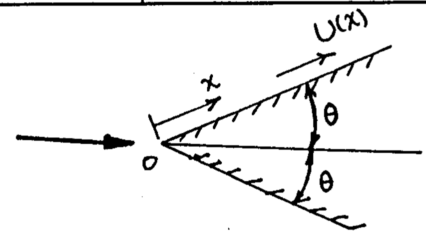

Figure 2.

3. Determine the conditions for dynamic similarity of an incompressible fluid flow inside two infinite parallel plates under the influence of a harmonically varying pressure gradient in the x-direction, governed by

$$\rho\frac{\partial u}{\partial t} = X\cos(\omega t) + \mu\frac{\partial^2 u}{\partial y^2}$$

where X is the amplitude of the pressure gradient. The fluid oscillates harmonically with a frequency ω. (20%)

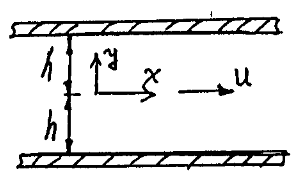

Figure 3.

4. A steady, incompressible, frictionless, two-dimensional jet of fluid with density ρ, breadth h, velocity V, and unit width impinges on a plate held at an angle α to its axis. Gravitational forces are to be neglected.
   (a) Determine the total force on the plate, and the breadths a, b of the two bransches.
   (b) Determine the distance ℓ to the center of pressure along the plate from the point O. The center of pressure is the point at which the plate can be balanced without requiring an additional moment. (30%)

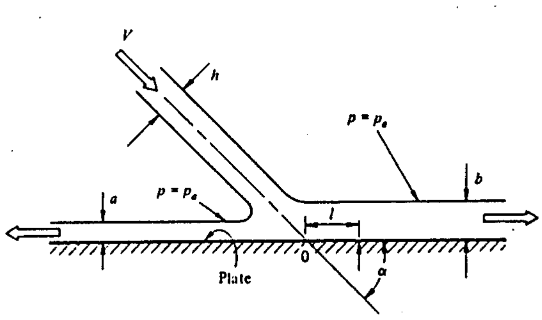

Figure 4.