(1). A stream of liquid of diameter d drains from a circular tank of diameter D as is shown in Fig.1. The depth of the water was h_0 at time t=0. Determine the water depth as a function of time, h=h(t).

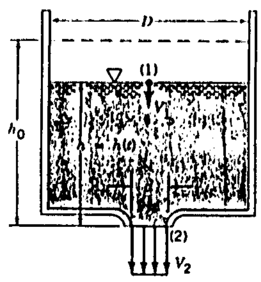
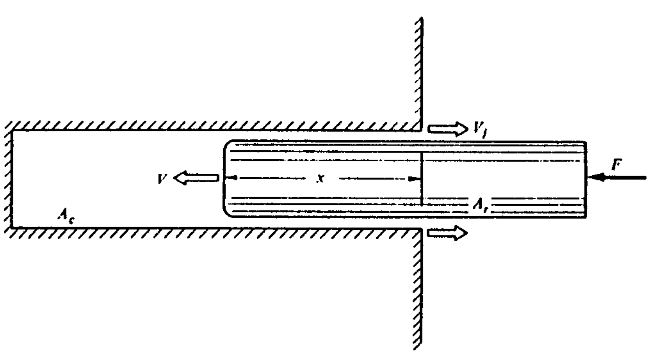


FIGURE 1

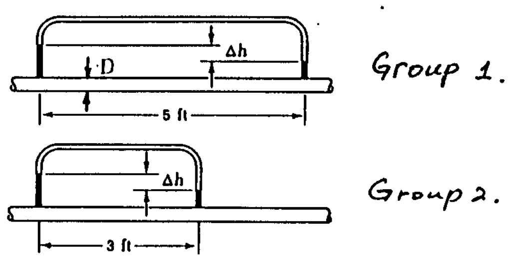
(2). A frequently used hydraulic brake consists of a movable ram that displaces water from a slightly larger cylinder as shown in the accompanying sketch. The area of the cylinder is A_C and the cross-sectional area of the ram is A_r. The ram velocity V does not change with time. Assume that the gap between the cylinder and the ram is much smaller than the displacement of the ram x. Use a coordinate system in which the ram is stationary.

(a) Determine the pressure at the end of the cylinder (where the velocity is assumed to be zero) and the jet velocity V_j.

(b) Find the force F on the ram in terms of A_r, A_C, and V. Assume that the cylinder is initially full of water and that gravity effects are negligible.



(3). This problem presents data from an actual experiment and then correlates them by using dimensional analysis. Consider a tube with inside diameter of 0.545 inch through which water flows. The tube is placed in a horizontal position, and pressure taps (迴路) are connected to a manometer that gives the pressure drop in terms of a head loss as water flows through the pipe. Head loss increases with flow rate. The test setup is shown in Figure 3. This head loss is due to friction between the liguid and the pipe wall. Data taken directly are given in columns 1, 2, and 3 of Table 1. Determine the appropriate formula to express the head loss Δh in terms of the relevant parameters.



Δh = head loss in the pipe (related to pressure drop by $\Delta h = \Delta p/\rho g$)
V = velocity of water in the pipe
ρ and μ = density and viscosity of the water, respectively
D = pipe diameter
L = length between pressure taps (5 ft, 3 ft)

TABLE 1 Results of pressure drop in a pipe experiment

Test	Q (gpm)	Δh (in.)	V (ft/s)	$\rho VD/\mu$	L/D	Δh/L	$(\Delta h/L)(\rho VD/\mu)^{-2}$
Group I							
1	1.50	2.50	2.06	9 097	110	0.0417	5.04×10^{-10}
2	1.85	3.69	2.54	11 220		0.0615	4.89
3	2.00	4.38	2.75	12 130		0.073	4.97
4	2.45	6.25	3.37	14 859		0.104	4.71
5	2.80	8.88	3.85	16 982		0.148	5.13
6	3.30	11.75	4.54	20 014		0.196	4.89
7	3.50	13.13	4.81	21 227		0.219	4.86
8	3.90	16.13	5.36	23 653		0.269	4.81
9	4.10	17.81	5.64	24 866		0.297	4.80
Group II							
11	1.50	1.50	2.06	9 097	66.1	0.0416	5.04×10^{-10}
12	1.85	2.20	2.54	11 220		0.0611	4.89
13	2.00	2.63	2.75	12 130		0.0731	4.97
14	2.45	3.80	3.37	14 859		0.106	4.71
15	2.80	5.33	3.85	16 982		0.149	5.13
16	3.30	7.00	4.54	20 014		0.194	4.89
17	3.50	7.88	4.81	21 227		0.219	4.86
18	3.90	9.70	5.36	23 653		0.269	4.81
19	4.10	10.70	5.64	24 866		0.297	4.81
							average 4.90×10^{-10}

Note: Group I data for 5 ft length; group II data for 3 ft length; $T = 70\,°F$; $\nu = 10.3 \times 10^{-6}$ ft²/s; $D = 0.545$ in.; $A = 0.00162$ ft².

4. Consider a laminar, steady, constant-property boundary layer flow over a semi-infinite porous flat plate with zero pressure gradient. A similar fluid is injected into the flow through the plate at a velocity $v_w = x^{-0.5}$ normal to the plate.

(a) Write the appropriate formulation for this boundary layer flow problem. (5%)

(b) Derive the momentum integral equation for the problem. (15%)

(c) Give the physical significance for the displacement thickness.of the boundary layer flow (5%)

5. Consider the incompressible inviscid flow represented by a velocity potential $\phi = U\, r(1+R^2/r^2)\cos\theta - A/(2\pi)$, where U, R, and A are constants.

(a) Show that this represents the flow past a stationary circular cylinder of radius R due to a uniform flow of velocity U at infinity. (10%)

(b) Determine the locations of the stagnation points and sketch the streamlines outside the cylinder for the cases $A/(\pi RU) = 0$, 2, and 4. (5%)

6. Calculate the velocity profile for incompressible laminar flow in an annulus formed by two concentric circular pipes of radius a and b. The inner pipe is kept stationary and the outer pipe is moved at a constant speed U. There is no pressure gradient in the flow. (10%)