八十二

1. Given that $x^2y'' + 2xy' + \lambda y = 0$

   with $y(1) = 0,\ y(e^2) = 0$

   (a) Find the eigenvalues and eigenfunctions (8%)

   (b) Put the equation in self-adjoint form (2%)

   (c) Give an orthogonality relation (2%)

2. If the matrix A can be diagonalized, then $P^{-1}AP = D$ or $A = PDP^{-1}$

   Show that (6%)

$$e^{tA} = Pe^{tD}P^{-1}$$

3. Let C be the straight line segment from i to 2 + i

   Show that (8%)

$$\left|\int_c \mathrm{Ln}(z+1)dz\right| \le \log_e 10 + \frac{\pi}{2}$$

4. The steady-state temperature u(x,y) in a semi-infinite plate is determined from

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad 0 < x < \pi, \quad y > 0$$

$$u(0,y) = 0, \quad u(\pi,y) = \frac{2y}{y^4+4} \qquad y > 0$$

$$u(x,0) = 0, \quad 0 < x < \pi$$

   Use a Fourier transform and the residue method to show that (14%)

$$u(x,y) = \int_0^\infty \frac{e^{-\alpha}\sin\alpha\sinh\alpha x}{\sinh\alpha\pi}\sin\alpha y\,d\alpha$$

1. Solve the initial and boundary value problem (15%)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + e^{-t}\sin 3x \qquad \text{for } 0<x<\pi, \quad t>0$$

Subject to: $u(0,t)=0$, $u(\pi,t)=1$, $u(x,0)=f(x)$

2. Express the following integral in terms of Bessel functions (5%)

$$\int x^{-2} J_2(x)\,dx = ?$$

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1. 試求下列常微分方程式之一般解(General solution)。

$$y'' + x^2 y' - 2y = e^x \cos(x) \qquad (10\%)$$

2. 試求函數 $e^{-2t}\int_0^t e^{2\tau}\cos(3\tau)\,d\tau$ 之拉普拉斯轉換(Laplace transform)。 (5%)

3. 已知函數 $f(t) = t\,u(t-1)$，試求 $\frac{df(t)}{dt}$ 之拉普拉斯轉換。(5%)

4. 試求和曲面 $z = x^2 + 2y^2$ 相切於點 $(2, 1, 6)$ 之切平面(Tangent plane)。 (10%)

5. 已知 $\vec{F}$ 為一向量場(Vector field)。試証

$$\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla \cdot \vec{F}) - \nabla^2 \vec{F} \qquad (10\%)$$

其中 $\nabla^2 \vec{F} = \frac{\partial^2 \vec{F}}{\partial x^2} + \frac{\partial^2 \vec{F}}{\partial y^2} + \frac{\partial^2 \vec{F}}{\partial z^2}$。

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