1. The rotating cylinder of Figure. 1 is 30 cm in diameter and 42 cm high. It is filled to a depth of 35 cm with liquid. Determine the angular velocity required to spill liquid over the top.

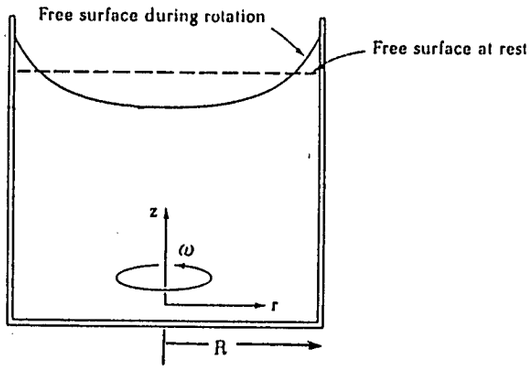

Fig. 1

2. A free jet in the form of a liquid sheet with area A strikes a flat plate inclined at an angle of θ as shown in Figure. 2 . The flow rate of the jet is Q. Calculate the force F_n and its point of application on the plate.

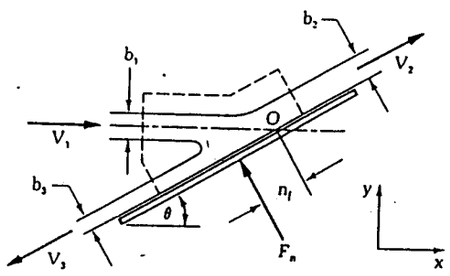

Fig. 2

3. A model is to be used to determine the velocity, V, of liquid flow through a small-diameter passage in a wall separating two pressurized tanks as shown in Fig. 3 Prototype characteristics are indicated on the figure. The model is to have a length scale of $\frac{1}{4}$, a viscosity scale of 2.0, and if possible, the pressures are to be the same for model and prototype. Assume that V is a function of $p_1 - p_2$, d, ℓ, and the fluid viscosity, μ. Determine the required dimensions for the model, and the velocity scale.

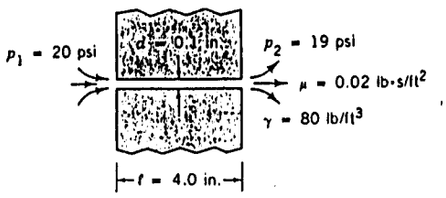

Fig. 3

4. It is often assumed that sharp objects can cut through the fluid better than blunt ones. Accordingly, the drag on the object shown in the sketch should be less when the fluid flows from the right to left than when it flows from left to right. However, experiments show that the opposite is true. Explain. (10%)

5. A R&D group was interesting in designing an axial-flow air compressor with rotor diameter of 0.6 m and an angular speed of 8000 rpm. Before deciding on the blade shapes, model tests were proposed. The model finally selected was o.3 m in diameter and was to pump water (ρ = 998 kg/m^3, ν = 1.1x10^{-3} m^2/s) rather than air. The properties of air are ρ = 1.0 kg/m^3, ν = 1.7x10^{-5} m^2/s.
   (a) For similar flows, what has to be the speed of the model? (10%)
   (b) For similar flows in two machines, what are the ratios of the head produced and the power required? Any possible effects of compressibility should be neglected. (10%)

6. Consider the steady laminar viscous fully-developed flow in a duct with an elliptical cross-section in the y-z plane normal to the flow direction. The elliptical cross-section can be described by the equation

$$\frac{y_b^2}{a^2}+\frac{z_b^2}{b^2}=1$$

where y_b and z_b denote boundaries of the pipe cross-section; a is the semimajor axis and b is the semiminor axis of the pipe. The fully-developed velocity profile can be assumed to be of the form:

$$u(y,z)=U_o(Ay^2+By+C)$$

where A, B, and C are constants to be determined. Obtain an expression of the friction factor for a given flow rate. (20%)