

1. 試用拉氏轉換法 (Laplace Transform) 解初始值問題。

$$xy'' + 2y' + (2-x)y = 2e^x$$

$$y(0) = 0.$$

(10%)

2. (a). 求逆拉氏轉換 (Inverse Laplace Transform)

$$\mathcal{L}^{-1} \left[ \frac{1}{s^2} (1 - \exp(-s))^2 \right] = ?$$

(6%)

(b) 函數  $f(t)$  可做 Laplace Transform 的條件是什麼? (2%)

$$(c) \mathcal{L}[f'(t)] = s \mathcal{L}[f(t)] - f(0^+)$$

能成立時,  $f(t)$ ,  $f'(t)$  需具備什麼條件? (2%)

3. 試解偏微分方程式

$$\frac{\partial^2 \theta}{\partial t^2} = c^2 \frac{\partial^2 \theta}{\partial x^2}$$

$$0 < x < L, t > 0.$$

$c^2$  為常數。

其邊界條件:  $\theta(0, t) = 0$        $\theta(L, t) = 0$        $t > 0$

起始條件:  $\theta(x, 0) = 0$        $\frac{\partial \theta}{\partial t} \Big|_{t=0} = \omega_0 x / L$        $0 < x < L$

(10%)

4. 試求矩陣  $[A]$  之特徵值及其對應之特徵向量

$$[A] = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

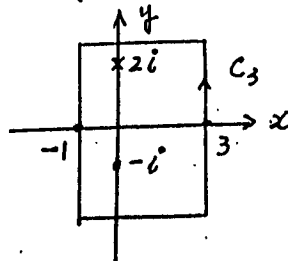
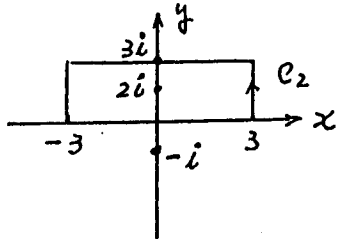
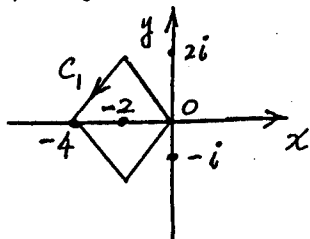
(12%)

5. 試將函數  $F(x_1, x_2) = 3x_1^2 + 3x_2^2 + 2x_1x_2$  轉換成標準式 (Canonical form).

(16%)

6. 試求出  $I_j = \oint_{C_j} \frac{\sin(z)}{z^2 - iz + 2} dz$  之積分值

其中  $C_j$  為  $C_1$ ,  $C_2$ , 和  $C_3$  並分別對應下列的圖形。(12%)



7. Solve  $xy'' + 2y' = 4x^3$  (10%)

8. Show that the vector field

$$\vec{F} = (y^2 \cos x + z^3) \vec{i} + (2y \sin x - 4) \vec{j} + (3xz^2 + 2) \vec{k}$$

is conservative, and deduce the corresponding scalar potential  $\Phi$  (20%)