

1. Solve $y'' + \exp(2x)y = \exp(3x)$ (10%)
2. If the Fourier coefficients of a periodic function $f(x)$ are a_n ($n = 0, 1, 2, \dots$) and b_n ($n = 1, 2, \dots$), what are the Fourier coefficients, say A_n, B_n , of the periodic function $f(x-a)$? (10%)
3. Let $\vec{v} = rz\vec{e}_r - 3\vec{e}_\theta + rz^2\vec{e}_z$. S is the part of the cylindrical surface that lies in the region $0 \leq r \leq 2, 0 \leq z < 6$, excluding the top surface. \vec{n} denotes the outward unit normal on S . Evaluate the surface integral

$$\iint_S \vec{v} \cdot \vec{n} dA \quad (13\%)$$

4. Solve the Sturm-Liouville problem

$$[x^3 y']' + (\lambda + 4)x^{-5}y = 0; \quad y(1) = y(e^2) = 0. \quad (10\%)$$

5. (a) Find $\mathcal{L} \left\{ \int_2^x u^2 e^{3u} du \right\}$. (5%)
- (b) Find the inverse Laplace transform of the given function e^{-4s}/s^2 . (5%)

6. Solve the differential equation $u_{xx} - u_{xy} - 2u_{yy} = 0$. (13%)

7. (12%) Assume that $f(z)$ is analytic on and within a closed contour C and that the point z_0 is within C , show that

$$\oint_C \frac{f'(z)}{z - z_0} dz = \oint_C \frac{f(z)}{(z - z_0)^2} dz$$

8. (12%) Prove that $AA^T = O$ implies that $A = O$, where A is any matrix and O is a matrix with zero elements.
9. (10%) Prove that the inverse matrix of A exists if and only if any eigenvalue λ_j of A is non-zero ($j=1, \dots, n$).