

1. Find the general solution of  $xy' - y = \frac{y}{\ln(y) - \ln(x)}$  (12%)

2. Evaluate  $\oint_{\Gamma} e^{1/z} dz$

if  $\Gamma$  is a closed path enclosing the origin. (13%)

3. Find the equation of the tangent plane and normal line to the surface  $2x = 3 + \cos(xyz)$  at the point  $(1, \pi, 1)$ . (12%)

4. Find the flux of  $\vec{F} = xz\vec{i} - y\vec{k}$  across the part of the sphere  $x^2 + y^2 + z^2 = 4$  lying above the plane  $z = 1$ . (13%)

5. (a) Consider the differential equation

$$(1-x^2)y'' - 2xy' + \lambda y = 0,$$

in which  $\lambda$  is a constant and  $-1 \leq x \leq 1$ . Does the above equation constitute a Sturm-Liouville problem?

(b) We can use the convolution theorem to show that the Laplace transform of  $\int_0^t f(\tau) d\tau$  is equal to  $(1/s)F(s)$ . Here,  $F(s)$  is the Laplace transform of  $f(t)$ . What assumptions are needed about  $f(t)$ ? (10%)

6. Determine all possible  $2 \times 2$  orthogonal matrices. (10%)

7. Let  $f(x) = e^{-kx}$  for  $x \geq 0$ , with  $k$  a positive constant. Answer the following.

(a) Is the Fourier cosine integral representation of  $f(x)$  equal to  $f(x)$  for all  $x \geq 0$ ? Why?

(b) Is the Fourier sine integral representation of  $f(x)$  equal to  $f(x)$  for all  $x \geq 0$ ? Why?

(c) Is the coefficient  $A(\omega)$  in the Fourier cosine integral of  $f(x)$  equal to the product of a constant and the Laplace transform of  $\cos(\omega x)$ ? Why? (15%)

8. Solve the problem:

$$\frac{\partial h}{\partial t} = a^2 \left( \frac{\partial^2 h}{\partial x^2} - \frac{\eta}{L} \frac{\partial h}{\partial x} \right) \quad (0 < x < L, t > 0),$$

$$h(0,t) = h(L,t) = 0 \quad (t > 0),$$

$$h(x,0) = \frac{KL}{a^2 \eta} [1 - e^{-\eta(1-x/L)}] \quad (0 \leq x \leq L). \quad (15\%)$$