1. Consider the system shown in Figure 1.
   a) What is the steady state error to a unit ramp input? (5%)
   b) The actual steady state error may be driven to zero for a unit ramp input by adding an element to the system as shown in Figure 2. For what value of k will the steady state error be zero? (5%)

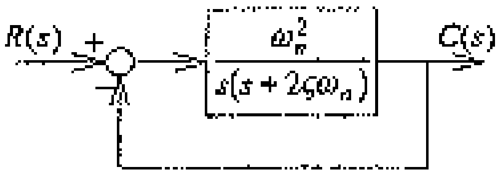

Figure 1

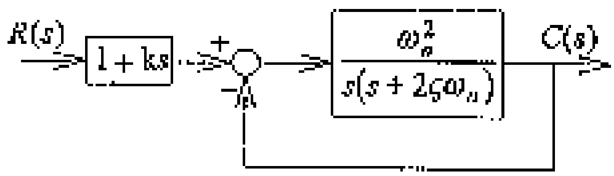

Figure 2

(10%) 2. Consider a feedback control system sketched in Figure 3.

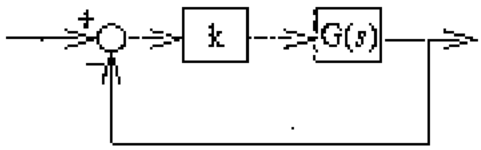

Figure 3

$$G(s)=\frac{b_0S^m+b_1S^{m-1}+\cdots+b_m}{S^n+a_1S^{n-1}+\cdots+a_n}\quad , \quad m\le n \quad (b_0>0)$$

Find conditions to guarantee the asymptotic stability (in the mathematical sense) of the feedback system for $k\to\infty$.

3. A feedback control system is to be designed to satisfy the following specifications:
   i) steady state error for a ramp input $\le 10\%$.
   ii) damping ratio of dominant poles ≥ 0.707.
   iii) settling time (5% criterion) of the system ≤ 3 seconds.

   The structure of the feedback control system is shown in Figure 4 where the amplifier gain, k_1, and the derivative feedback gain, k_2, are to be selected.

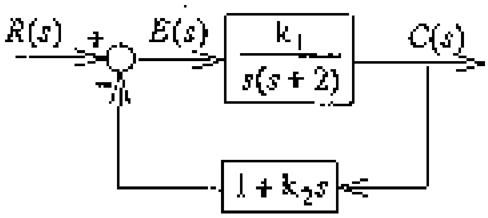

Figure 4

   a) What is the minimum value of the amplifier gain, k_1? (5%)
   b) Sketch the S-plane indicating the region in which the closed loop poles must lie to meet the above specifications. (5%)
   c) Sketch the root locus for k_1 given a value of k_2 in the range:
      $0.5<k_2<\infty$ (Note: $k_2\ne 0.5$)
      Just indicate the shape (any breakaway and arrival point values need not be derived) and direction of increasing k_1. (5%)
   d) Do the same for $k_2<0.5$. (5%)
   e) What must the minimum value of the derivative feedback gain, k_2, be in order to meet the above specifications? (10%)

4)

A linear system is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 3 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

1a) Find the irreducible transfer function, from u to y, of the above system? (8%)

1b) Is the system completely state controllable and/or completely observable? Why? (5%)

5)

For the controlled plant

$$G(s) = \frac{s-1}{(s+4)(s-5)}$$

please find a controller $C(s)$ such thathe closed-loop transfer function of the unity feedback system is given by (6%)

$$G_f(s) := \frac{G(s)C(s)}{1+G(s)C(s)} = \frac{1}{s+5}$$

Is the closed-loop system asymptotically stable (why) ? (6%)

6) (10%)

Determine, by using the Nyquist stability criterion, for what values of k the negative unity feedback system with the following open-loop transfer function is stable

$$G(s) = \frac{k}{s(2s+2)(2s+1)}$$

7)

Find the transfer function, $\frac{V_2(s)}{V_1(s)} = \alpha\frac{\tau_1 s+1}{\tau_2 s+1}$, of the circuit shown in Fig. 5 (7%),
and show that the circuit with $R_1 = 9k\Omega$, $R_2 = 100k\Omega$, $R_3 = 1k\Omega$ and $C = 1000\mu F$ is a phase-lead network from the Bode plot (only approximate plot, but having information of α, τ_1 and τ_2 in the plot) (8%).

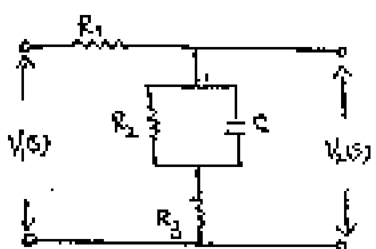

Fig.5