1. (以下各小題為是非題。作答時，各小題題號應標示清楚。答錯倒扣，請依順序以"是"、"非"或空白作答；倒扣時，計分至本大題為零分為止。)

(a) $\int xdy$ around a closed curve gives the area inside. (6%)

(b) If du/dt decays expoentially, then $sU(s) \to u(\infty)$ as $s \to \infty$ with $U(s)$ denoting the Laplace transform of $u(t)$. (6%)

(c) The inverse Laplace transform of $\frac{1}{s(s-4)^2}$ is $\frac{1}{4}te^{4t} - \frac{1}{16}e^{4t} + \frac{1}{8}$. (6%)

(d) In general, any linear, second-order, differential equation

$\frac{d^2y}{dx^2} + R(x)\frac{dy}{dx} + (Q(x) + \lambda P(x))y = 0$ can always be transformed into Sturm-Liouville form. (6%)

(e) $u(t)$ is the solution of $du/dt = u^{1-k}$ with $u(0) = 1$ and $k \neq 0$. Then, $u(t)$ blows up at $t = -1/k$ for $k < 0$. (6%)

(f) Let φ and its first partial derivatives be continuous for all (x,y) in D and let $\bar{F} = \nabla\varphi$. Then $\int_C \bar{F} \cdot d\bar{r} = 0$, where C is a closed path in D. (6%)

2. Inspection shows that $(x^2-1)y'' - 2xy' + 2y = 0$ has $y_1 = x$ as a first solution. Find another independent solution $y_2(x)$ by the method of reduction of order. (14%)

7311

88 學年度 國立成功大學 碩士班招生考試 機械 系所 工程數學 試題

3. (12%) Solve for the following two systems by matrix method

(a) $my'' + cy' + ky = 0$, where $m=1$, $c=2$, $k=0.75$

(b) $\begin{cases} x_1' = 3x_1 + 3x_2 + 8 \\ x_2' = x_1 + 5x_2 + 4e^{3t} \end{cases}$

4. (12%) Solve for the boundary value problem

$$\begin{cases} \dfrac{\partial u}{\partial t} = a^2 \dfrac{\partial^2 u}{\partial x^2} & (0 < x < L,\ t > 0) \\ u(0,t) = 0, \quad \dfrac{\partial u}{\partial x}(L,t) = -A\,u(L,t) & (t \geq 0) \\ u(x,0) = f(x) & (0 < x < L) \end{cases}$$

5. (13%) Let $f(x) = e^{-|x|}$, Compute the Complex Fourier Integral of f

Note that $f(x) = \begin{cases} e^{-x}, & \text{if } x \geq 0 \\ e^{x}, & \text{if } x < 0 \end{cases}$

6. (13%) Let $\Gamma_1(x) = 3e^{it}$ for $0 \leq t \leq \frac{\pi}{2}$ and $\Gamma_2(x) = t^2 + 3(t+1)i$ for $0 \leq t \leq 1$

The curve of Γ_1 is a quarter-circle of radius 3 about the origin from 3 when $t=0$ to $3i$ when $t=\frac{\pi}{2}$. The curve Γ_2 extends from $3i$ when $t=0$ to $1+6i$ when $t=1$. please evaluate $\int_{\Gamma} I_m(z)\,dz$.