(一)

(10%) A boy, standing in corner A of a rectangular pool, has a boat in the adjacent corner B on the end of a 10-meter-long string. He walks along the side of the pool toward C keeping the string taut. Locate the boy and boat when the latter is 6 metres from AC. **[Hint:** Choose the coordinate system so that AC is along the x-axis and AB is along the y-axis].

Given that : $\ell n(3) \approx 1.1$

(15%) Solve

$$u_t(x,t) = \kappa u_{xx}(x,t) + F(x,t) \quad 0 < x < \ell,\ t > 0$$
$$u(x,0) = f(x) \quad 0 < x < \ell$$
$$u(0,t) = u(0,\ell) = 0 \quad t > 0$$

(二) Find solutions for the following integral and differential equations

(a) $I = \int_1^2 x^{-3} J_4(x)\,dx$; where $J_3(1) = 0.0196$, $J_3(2) = 0.1289$ (5%)

(b) $x(x-1)y'' - xy' + y = 0$ (10%)

(c) $y_1' - 4y_1 - y_2 = 0$
$y_2' + y_1 - 2y_2 = 0$ (10%)

(三)

(a) (5 %) Prove $\iiint_V \nabla \times \mathbf{B}\,dV = \iint_S \mathbf{n} \times \mathbf{B}\,dS$, in which V is the volume bounded by a closed regular surface S, $\mathbf{n}$ is the outward unit vector normal to S, and $\mathbf{B}$ is a continuously differentiable vector field.

(b) (10 %) Let $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Find $e^{\mathbf{A}t}$

(c) (10%) Evaluate $\int_0^\infty \frac{\sin^2 x}{x^2}\,dx$ using complex contour integral.

(背面仍有題目,請繼續作答)

(四) Laplace Transform and Fourier Analysis (25 points)

For problem 3(a), 3(b), and 3(c), please just select the right answer and fill in the answer sheet.
For problem 3(d), please provide brief description.

3(a) Assume F(s) is the Laplace Transform of the function

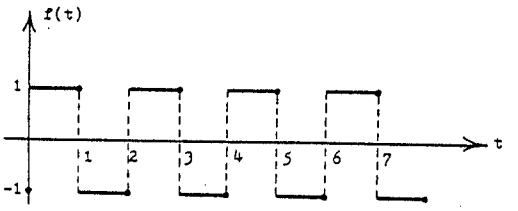
$$f(t)=e^{-t}t\cos 2t$$

What is F(3)?

(a) 1/49; (b) 3/100; (c) 5/24; (d) 7/120; (e) 5/36 **(7 points)**

3(b) Find the Laplace transform L(f(t)), where f(t) is the square wave shown in the following figure.

(a) $\frac{1}{s}\cos\frac{s}{2}$, (b) $\frac{1}{s}\cot\frac{s}{2}$, (c) $\frac{1}{s}\tanh\frac{s}{2}$, (d) $\frac{1}{s}\sin\frac{s}{2}$, (e) $\frac{1}{s}\cosh\frac{s}{2}$ **(7 points)**



3(c) Find the solution of the system

$$\frac{dx}{dt}-6x+3y=8e^{t}$$
$$\frac{dy}{dt}-2x-y=4e^{t}$$

subject to the initial conditions

$$x(0)=-1,\ y(0)=0$$

What is X(s)? **(4 points)**

(a) $\frac{-3}{s+4}+\frac{2}{s-1}$; (b) $\frac{1}{s-4}-\frac{2}{s-1}$; (c) $\frac{-3}{s+4}+\frac{2}{s+1}$; (d) $\frac{2}{s^2+4}-\frac{3}{s+1}$; (e) $\frac{3}{s^2+4}+\frac{1}{s-1}$

What is the value of x(t)-3y(t) **(3 points)**

(a) e^{t}; (b) $-e^{4t}$; (c) $-3e^{4t}$; (d) $-3e^{t}$; (e) $-e^{2t}$

3(d) Please briefly describe when you should use Fourier series; Fourier Integral; and Fourier Transform? **(4 points)**