Problem 1 (18 points)

Oil spills may occur in ports where oil tankers are loaded. The density of oil ρ_O is less than that of water ρ_W, and the two fluids are immiscible, so that when a spill occurs, the oil simply spreads out in a layer on top of the water.

To contain any possible spills, a semi-circular "oil boom" is deployed at a radius R around the dock where the loading takes place. The boom is a barrier which floats on the water, its bottom submerged and its top a bit above the water surface, as shown in Fig. 1. This barrier prevents the oil from spreading past it, at least if the spill is not too great.

Suppose a volume V of oil is spilled inside the boom. After sufficient time has elapsed for the situation to reach static conditions, calculate, in terms of ρ_O, ρ_W, R, V, and the gravitational acceleration g,

(a) the depth h_1 of the bottom surface and the elevation h_2 of the top surface of the contained oil relative to the water surface outside the boom;

(b) the components of force parallel to and transverse to the dock exerted by one of the moored boom ends on the dock.

Problem 2 (24 points)

A rainstorm hits a flat, horizontal roof. The rain pours down at a mass flow rate $\dot{m}$ per unit horizontal area, each drop moving with a speed V and at an angle θ relative to the vertical, as shown in Fig. 2. Soon a steady state is established where the water of density ρ is driven rightward over the roof in a thin layer and over the edge at right, while the raindrops splatter violently onto the top part of the water layer.

(a) If the slope of the water surface is very small, what is the pressure distribution in the water layer in the y-direction, perpendicular to the roof surface? Is the pressure just below the water surface atmospheric? Explain.

(b) If friction between the water and the roof is negligible, derive an expression for the depth $h(x)$ of the water at x. Assume that $h \ll x$, and that $h \approx 0$ at $x \approx 0$. The given quantities here are $\dot{m}$, V, θ, ρ, g.

(c) If friction at the roof were not negligible, would $h(x)$ increase or decrease? Explain.

Problem 3 (18 points)

A metal ball falls at steady speed in a large tank containing a viscous liquid. The ball falls so slowly that it is known that the inertia forces may be ignored in the equation of motion compared with the viscous forces.

(a) Perform a dimensional analysis of this problem, with the aim of relating the speed of fall V to the diameter of the ball D, the mass density of the ball ρ_b, the mass density of the liquid ρ_1, and any other variables which play a role. Note that the "effective weight" of the ball is proportional to $(\rho_b - \rho_1)g$.

(b) Suppose that an iron ball (SG (specific gravity) = 7.9, D = 0.3 cm) falls through a certain viscous liquid (SG = 1.5) at a certain steady-state speed. What would be the diameter of an aluminum ball (SG = 2.7) which would fall through the same liquid at the same speed, assuming inertia forces are negligible in both flows?

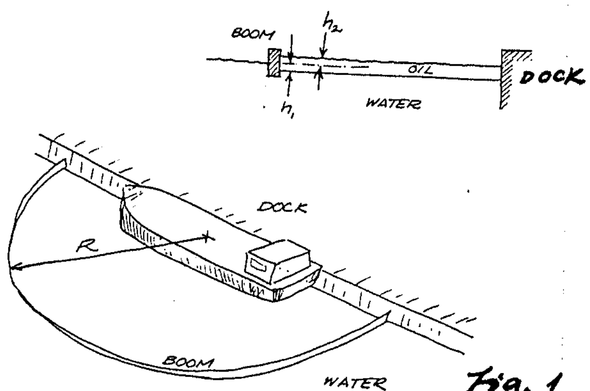

Fig. 1

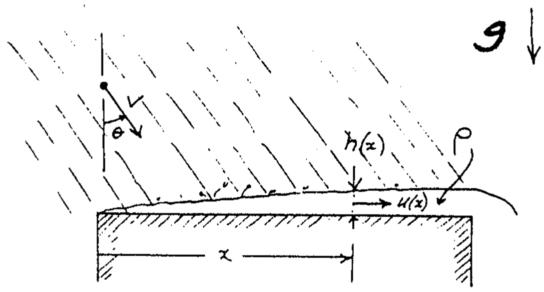

Fig. 2

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Problem 4 (point 25)

Consider turbulent flow of an incompressible fluid past a flat plate. The boundary layer velocity profile is assumed to be $u/U = (y/\delta)^{1/7} = Y^{1/7}$ for $Y = y/\delta \leqq 1$ and $u = U$ for $Y > 1$ as shown in Fig. 4. This is a reasonable approximation of experimentally observed profiles, except very near the plate where this formula gives $\partial u/\partial y = \infty$ at $y = 0$. Note the differences between the assumed turbulent profile and the laminar profile. Also assume that the shear stress agrees with the experimentally determined formula:

$$\tau_w = 0.0225 \rho U^2 \left(\frac{\nu}{U\delta}\right)^{1/4}$$

Determine the boundary layer thickness δ, δ^*, and θ and the wall shear stress, τ_w, as a function of x.

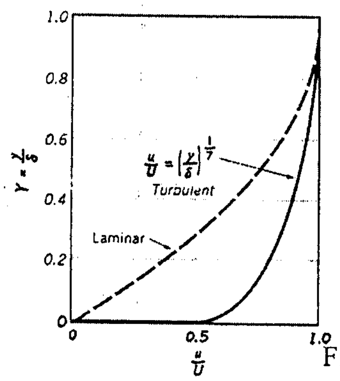

Figure 4

Problem 5 (point15)

Ethylene glycol at 60℃, with a velocity of u_m = 4cm/s, enters the 6-m-long, heated section of a thin-walled, 2.5-cm-ID tube, after passing through an isothermal calming section. In the heated part, the tube wall is maintained at a uniform temperature T_w =100℃ by condensing steam on the outer surface of the tube. Calculate the exit temperature of ethylene glycol. The physical properties of the fluid at the inlet temperature 60℃ are: specific heat Cp = 2562 J/(kg.℃), density ρ = 1088 kg/m^3, kinematics viscosity ν = 4.75 ×10^{-6} m^2/s, thermal conductivity k = 0.26 w/(m.℃), Pr =51.