(甲,乙,丙,丁,戊組)

91 學年度	國立成功大學 碩士班招生考試	機械研究 系所	工程數學	試題	共 2 頁 第 1 頁

1. a) 請定義何謂自伴微分演算子(Self-adjoint differential operator)? 並列出正則史數-利奧比系統(Regular Sturm-Liouville system)微分方程式和其邊界條件?(4%)

b) 正則史數-利奧比系統微分方程之解,具何種性質? 請說明(不必証明),並定義何謂正交函數系統之完整性條件(Complete orthogonal system)? (8%)

c) 求微分方程

$$\frac{d^2y}{dx^2} + \lambda y = 0 \quad , \quad \text{其邊界條件 } y(0)=0, \quad y(l)=0$$

証明其特徵解在 $[0, l]$ 之間為正交函數?

如果為正交函數,請以此正交函數系統

將函數 $f(x)=x^2 \quad 0 \le x \le l$. 展開為 Fourier Series.

(13%)

2-1. Evaluate the following integrals: (8%)

(a) $\int_0^\infty (x+1)^2 e^{-x^3} dx$

(b) $\int_0^\infty \frac{x^c}{c^x} dx$

2-2. Let $f(x) = x$ for $0 \le x \le 1$.

(a) Expand $f(x)$ in a Fourier cosine series for period 2. (6%)

(b) Expand $f(x)$ in a Fourier sine series for period 2. (6%)

(c) Explain the relation of the solutions obtained from (a) and (b). (5%)

3、求解下列聯立方程組 (25%)

$$\begin{cases} x_1 - x_3 + 2x_4 + x_5 + 6x_6 = -3 \\ x_2 + x_3 + 3x_4 + 2x_5 + 4x_6 = 1 \\ x_1 - 4x_2 + 3x_3 + x_4 + 2x_6 = 0 \end{cases}$$

4-1. (10%) Classify the singularities of $f(z) = \dfrac{1}{(z-1)(z-2)}$. Obtain the Laurent expansion centered on $z = 0$ for the three regions: (i) $|z| < 1$, (ii) $1 < |z| < 2$, (iii) $|z| > 2$.

4-2. (15%) Sometimes it is possible to find a physically interesting solution to a partial differential equation by assuming that the solution is a function of a single variable rather than two or more variables. In the particular case of the heat conduction equation

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$$

try a solution of the form $u(x,t) = f(\xi)$ with $\xi = xt^{-\alpha}$. For what value of α does this work, and what is the result for $f(\xi)$ in that case?