Problem 1. (25 points)

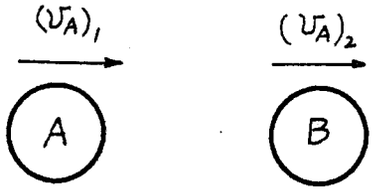
(a) Consider the problem of central impact between two particles A and B having mass m_A and m_B, respectively. If $(v_A)_1$ and $(v_B)_1$ are the initial velocities of particles A and B in the same direction before impact, respectively, where $(v_A)_1 > (v_B)_1$, $\int P\mathrm{d}t$ and $\int R\mathrm{d}t$ denote the deformation impulse and restitution impulse, respectively, $(v_A)_2$ and $(v_B)_2$ are the final velocities of particles A and B after impact, respectively,

(i) Show that: the momentum of the system of two particles is conserved before and after the impact. (5 points)

(ii) If the coefficient of restitution, e, is defined as the ratio of restitution impulse to deformation impulse, i.e., $e = \int R\mathrm{d}t / \int P\mathrm{d}t$, show that: $e = [(v_B)_2 - (v_A)_2]/[(v_A)_1 - (v_B)_1]$. (5 points)

(iii) Show that: for the case of elastic impact, the total energy of the system is conserved before and after the impact. (5 points)

(b) For 3-dimensional oblique impact between two particles A and B having mass m_A and m_B, respectively, as well as the given initial velocities of particles A and B before impact, please show the equations that are sufficient to solve the unknown velocities of A and B after impact. (Just only write down the equations required with no necessity to solve the unknowns.) (10 points)



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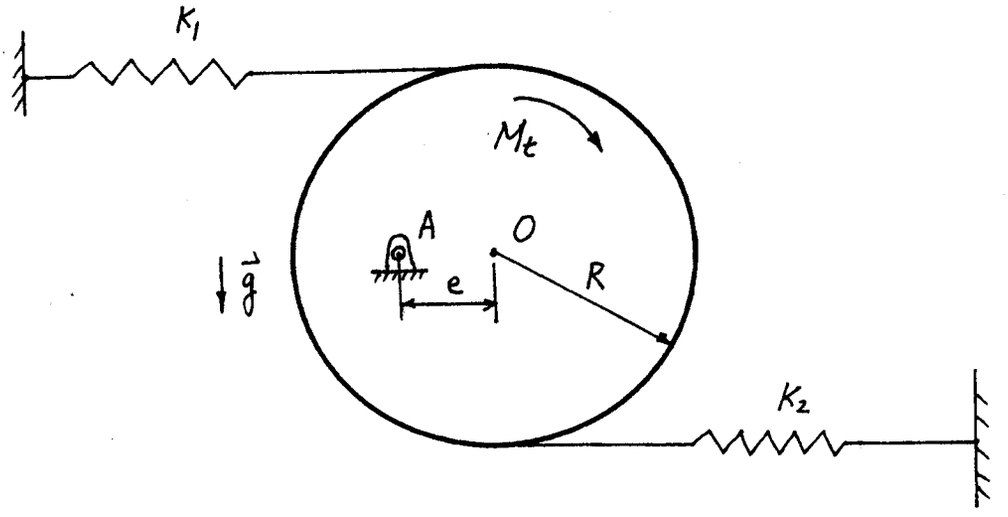
Problem 2. (25 points)

A homogeneous disk has a mass of m and a radius of R, and is rotating about point A (that is eccentric at a distance, e, from the disk center O) under the action of external moment, $M_t = M_0 \sin \omega t$. Two springs with spring constant k_1 and k_2 are originally unstretched,

(a) Write down the equations of motion of the system. (10 points)

(b) Determine the natural frequency and the amplitude of vibration. (8 points)

(c) Determine the condition of resonance of the system, and explain the phenomenon of resonance. (7 points)



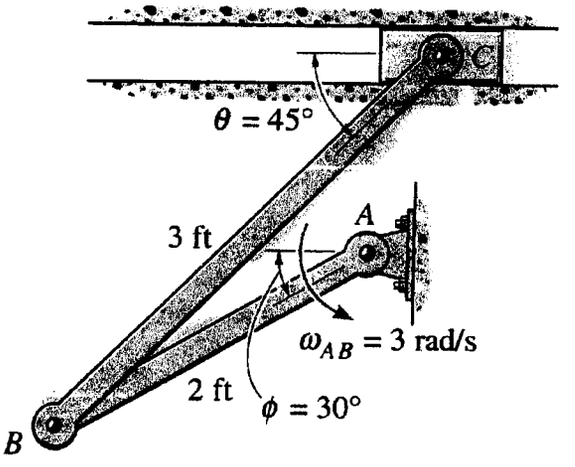
Problem 3. (25 Points)

(a) For a general relative motion. The absolute acceleration of a point on a rigid body can be decomposed into five terms. Please write down the expression and try your best to explain the physical meaning of each term. (5 Pts)

(b) A sphere, and a hoop, both have the same radius and mass, were released from rest on a incline at the same height. Which one will reach the bottom faster? Why? (5 Pts)

(c) Under what situation, the momentum conservation is valid? Under what situation, the system energy is conserved? (5 Pts)

(d) If the angular velocity of link AB is ω_{AB} = 3 rad/s. Where is the instantaneous center? (Please just sketched on the figure) (5 Points) In addition, please also determine the velocity of the block at C and the angular velocity of the connecting link CB at the instant θ=45° and ϕ=30°. (5 Points)



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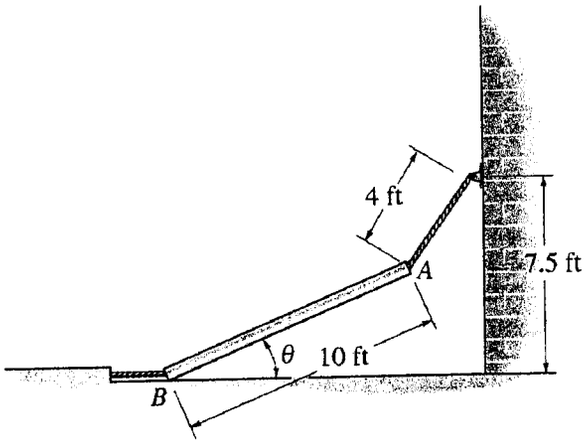
Problem 4. (25 Points)

(a) Is angular momentum parallel to angular velocities for a general 3D motion? Why? (4 Pts)

(b) In rolling without slipping situation, the friction force in rolling motion does or does no work? Why? (4 Pts)

(c) The slender beam having a weight of 200 lb is supported by two cables. If the cable at end B is cut so that the beam is released from the rest when θ=30°, determine the speed at which end A strikes the wall. Neglect friction at B. (9 Pts)



(d) The mass center of the *3-lb* ball has a velocity of $(v_G)_1 = 6\ ft/s$ when it strikes the end of the smooth *6-lb* slender bar. Determine the angular velocity of the bar about the z-axis just after impact if $e=0.7$. (8 Pts) Hint: Please treat the ball as a particle.