

本試題是否可以使用計算機: 可使用, 不可使用 (請命題老師勾選)

1. Please solve the following ordinary differential equation by the method of variation of parameters: (15%)

$$x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = x \log x$$

2. Determine the coefficients in the representation (15%)

$$f(x) = \sum_{n=1}^{\infty} A_n \sin nx \quad (0 < x < \pi)$$

In the following cases:

(a) $f(x) = 1$, (b) $f(x) = x$, (c) $f(x) = \begin{cases} 1 & (x < \pi/2) \\ \frac{1}{2} & (x = \pi/2) \\ 0 & (x > \pi/2) \end{cases}$

3. Find the value of the following infinite integration, (20%)

$$\int_0^{\infty} \frac{\cosh ax}{\cosh x} dx \quad \text{where } |a| < 1$$

(背面仍有題目, 請繼續作答)

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4. Find
- Fourier Transform,
 - Laplace Transform, and
 - Fourier Series

(assume period = $4a$, i.e. duty cycle = 50 %)

of $f(x)$, where

$$f(x) = \begin{cases} 1, & \text{if } -a < x < a \\ 0, & \text{otherwise} \end{cases}$$

(21 %)

5. Find a unit normal vector \bar{n} of the cone of revolution

$$z^2 = 16(x^2 + 4y^2) \quad \text{at the point P: (1, 0, 4).} \quad (14 \%)$$

6. A particle moves once counterclockwise about the rectangle

with vertices (1, 1), (1, 7), (3, 1) and (3, 7) under the

influence of the force \vec{f} , where

$$\vec{f} = \left[-\cosh(4x^4) + xy \right] \bar{i} + \left(e^{-y} + x \right) \bar{j}, \quad \text{where } \bar{i} \text{ and}$$

\bar{j} are the unit vectors in X-axis and Y-axis respectively.

Calculate the work alone.

(15%)