

1. Evaluate  $I = \oint_C \cot(z) dz$ , where  $C$  is the unit circle  $|z|=1$  traversed in a clockwise sense. (15%)

2. Let  $f(x) = x^2/2$  for  $-0 \leq x \leq \pi$ . Find the Fourier series of  $f(x)$  and evaluate the sum of the series  $\sum_{n=1}^{\infty} \frac{1}{n^2}$ . (15%)

3. Find the general solution of the equation  $\frac{d^2y}{dx^2} + 2\frac{dy}{dt} + 2y = \delta(x-3)$ . (20%)

4. Solve  $\dot{X} = AX$ , where  $X^T = [x_1 \ x_2]$ ,  $\dot{X} = dx/dt$ ,

$A = \begin{bmatrix} 1 & 3 \\ -3 & 7 \end{bmatrix}$ , the superscript  $T$  denotes transpose of a vector or matrix. (15%)

5. Let  $F(x, y, z) = (-y + z, x + yz, xyz)$ . By applying Stokes' Theorem, compute the integral of Curl  $F$  over the hemisphere  $x^2 + y^2 + z^2 = 1, z \geq 0$ , with outwards normals. (15%)

6. Find the general solution of  $y'' - \frac{4}{x}y' + \frac{4}{x^2}y = x^2 + 1$ ,  $x > 0$ ,

where  $y' \equiv dy/dx$ ,  $y'' \equiv d^2y/dx^2$ .

(20%)