1. (20 %) As illustrated in Fig. 1 below, a frictionless fluid of density ρ_1 enters a chamber where it is heated, so that the density decreases to ρ_2, while the pressure remains approximately constant. The light fluid then escapes through a chimney that has a height y. Except for the heating process, treat the fluid as incompressible. The velocity of the fluid entering the heating chamber is negligible, and the cold fluid, which also has a density ρ_1, completely surrounds the chimney.

(a) Using **dimensional analysis**, define a dimensionless velocity of the fluid in the stack, and determine the dimensionless group(s) upon which it depends.

(b) Now, **calculate** the velocity V of the fluid in the stack. Express your result in terms of ρ_1, ρ_2, g, and y. Is the result consistent with that of part (a)? **Hint:** At the ground level, the pressure of the hot fluid is approximately the same as that of the cold fluid outside the heating chamber. Assume also that the pressure of the cold fluid outside has a hydrostatic distribution.

2. (30 %) In an investigation of an incompressible turbulent boundary layer in a region of rapidly rising pressure, mean velocity distributions were measured at section 1 just upstream of the region of pressure rise and at section 2 just at the point of separation (see Fig. 2). Except for the laminar sublayer very close to the wall, where accurate measurements were not possible, the dimensionless velocity distributions at sections 1 and 2 were found to be represented by

$$\frac{u_1}{U_1} = \left(\frac{y}{\delta}\right)^{1/7} \quad \text{and} \quad \frac{u_2}{U_2} = 5\left(\frac{y}{\delta}\right)^4 - 4\left(\frac{y}{\delta}\right)^5,$$

respectively. The measurements showed further that the free-stream velocities and distances were related as follows:

$$U_2 = 0.90\,U_1, \quad \delta_2 = 4.00\,\delta_1, \quad \Delta x = 40.0\,\delta_1.$$

(a) Estimate the percentage increase in mass flow in the boundary layer between sections 1 and 2.

(b) Estimate the increase in wall static pressure, in terms of the dimensionless ratio $(p_2 - p_1)/(\rho U_1^2/2)$.

(c) Estimate the value of $\bar{\tau}_0/(\rho U_1^2)$, where $\bar{\tau}_0$ is the average skin-friction stress acting on the wall between sections 1 and 2. **Hint:** Carry out a momentum analysis for the rectangular control volume bounded by $x = x_1, x_2$ and $y = 0, \delta_2$ (as shown in Fig. 2). There clearly is a net mass flowrate across the control surface at $y = \delta_2$, throughout which the x-component of fluid velocity may be approximated by $(U_1 + U_2)/2$ (since U_1 and U_2 differ only slightly).

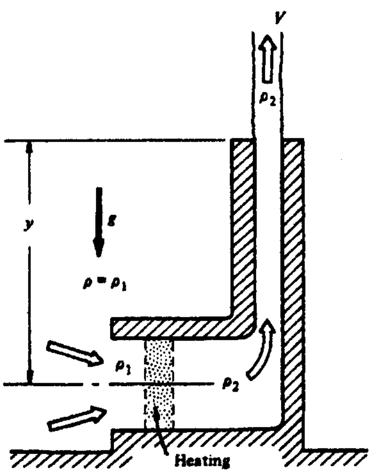

Fig. 1

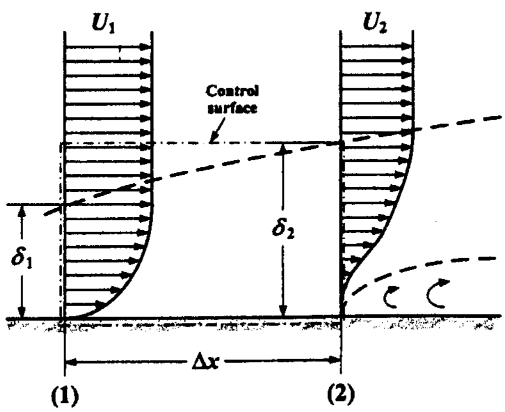

Fig. 2

3. **(10%)** Consider the flow of a fluid in which the fluid elements are traveling with velocity, u, in the x direction (this is the only non-zero velocity of the fluid necessary to consider in this problem). A succession of fluid elements travel through the Eulerian point $x = x_0$, with a velocity $u = u_0$ and subsequently accelerate according to

$$u = (u_0 / x_0)\, x$$

However the flow is **steady**. Chemical constituents within the fluid are reacting in such a way that the concentration, c, of one of the constituents is increasing with time at a rate denoted by α (a constant). If the concentration at the point $x = x_0$ has a known and constant value denoted by c_0 find an expression for the concentration elsewhere as a function of x, x_0, u_0, c_0, and α.

4. **(20%)** When a vehicle such as an automobile slams on its breaks (locking the wheels) on a very wet road it can "hydro-plane". In these circumstances a film of water is created between the tires and the road. Theoretically a vehicle could slide a very long way under these conditions though in practice the film is destroyed before such distances are achieved (indeed tire treads are designed to prevent the persistence of such films).

To analyze this situation consider a vehicle of mass, m, sliding over a horizontal plane covered with a film of liquid of viscosity, μ. Let the area of the film under all four tires be A and the film thickness (assume uniform) be h. If the velocity of the vehicle at some instant is u, find the force slowing the vehicle down in terms of A, u, h, and μ. Find distance, L, it would slide before coming to rest assuming that A and h remain constant (this is not, of course, very realistic). What is this distance for 1000 kg vehicle if $A = 0.1$ m^2, $h = 0.1$ mm, $u = 10$ m/sec and the water viscosity is 0.001 kg/m sec?

5. **(20%)** A constant and uniform layer of Newtonian, viscous, incompressible liquid (dynamic viscosity, μ, and density, ρ) flows down a flat plate inclined at an angle, θ, to the horizontal:

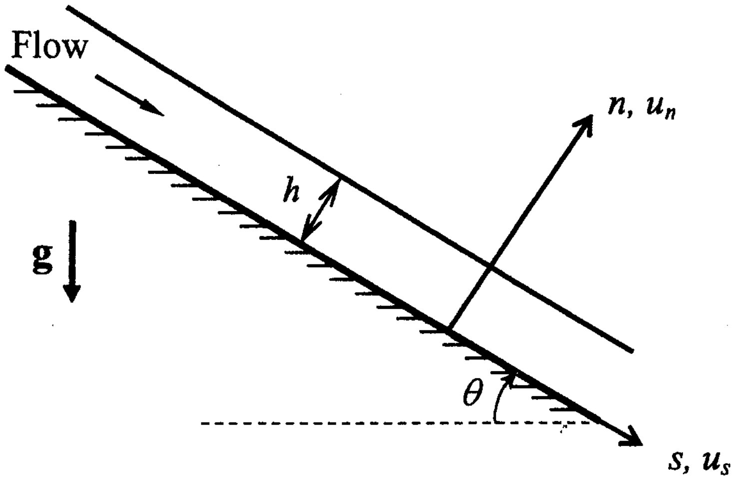

Fig. 3

The thickness of the layer is h and the flow is planar with velocity components as follows:

$$u_n = 0 \quad ; \quad u_s(n) = Cn(2h - n)$$

Find:

(a) An expression for C in terms of ρ, μ, θ and the acceleration due to gravity, g.

(b) The pressure acting on the plate if the atmospheric pressure is denoted by p_a.