系所組別： 機械工程學系甲、乙、丙、丁、戊組

考試科目： 工程數學

※ 考生請注意：本試題 □可 ☑不可 使用計算機

1. The equations that govern the reaction rates of a simple chemical reaction system, $A_1 \leftrightarrow A_2$, are $-\frac{d[A_1]}{dt} = k_1[A_1] - k_2[A_2]$, and $-\frac{d[A_2]}{dt} = -k_1[A_1] + k_2[A_2]$. [A1] and [A2] denote concentrations of species A_1 and A_2, respectively. k_1 is the forward reaction rate coefficient and k_2 is the rate coefficient for the reverse reaction. At time $t=0$, it is known that the concentration for A_1 equals to A_0, and no species A_2 exists. Please derive expressions for evolutions of species concentrations, $[A_1]$ and $[A_2]$, in terms of A_0, k_1, and k_2. Determine the concentrations of species A_1 and A_2 when the system reaches steady state. Show your work step by step. (10%)

2. Derive the solution of the following equation and boundary conditions:

$$f\frac{d\eta}{dx} = D\frac{d^2\eta}{dx^2} + (1-\eta)$$

$\eta(0) = \eta^*$ and $\eta(\infty) = 1$

f and D are constants, and $D > 0$. Your solution should be in terms of f, D, η^*, and x. (8%)

3. Find the general solution formula to the equation

$$\frac{d^3x}{dt^3} + 3\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + x = 0$$

Is the solution asymptotically stable? (7%)

4. Evaluate $\oint_C (y - \sin x)dx + \cos x dy$ where C is the triangle of the adjoining figure as shown in Fig. 1:

(a) directly (10%)

(b) by using Green's theorem in the plane. (15%)

5. If $f(t) = \begin{cases} 2, & -2 \le t < -1 \\ 1, & -1 \le t < 1 \\ 2, & 1 \le t \le 2 \end{cases}$,

(a). find the Fourier Series of $f(t)$. (10%)

(b). find the value of the Fourier Series, found in (a), converges to, when t is an integer. (5%)

(c). find the steady state solution of the O.D.E: $y'' + 25y = f(t)$, where $y'' \equiv d^2y/dt^2$. (10%)

(背面仍有題目,請繼續作答)

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6.

Find the Laurent expansion of the complex function $f(z)=\dfrac{z+2}{z^2+4}$ about the point $z=2i$. (15%)

Evaluate the integrals $\oint_C \dfrac{e^{2z}}{z^2(z-2z-3)}dz$ along the path C that is the counterclockwise circle with $z=|4|$. (10%)

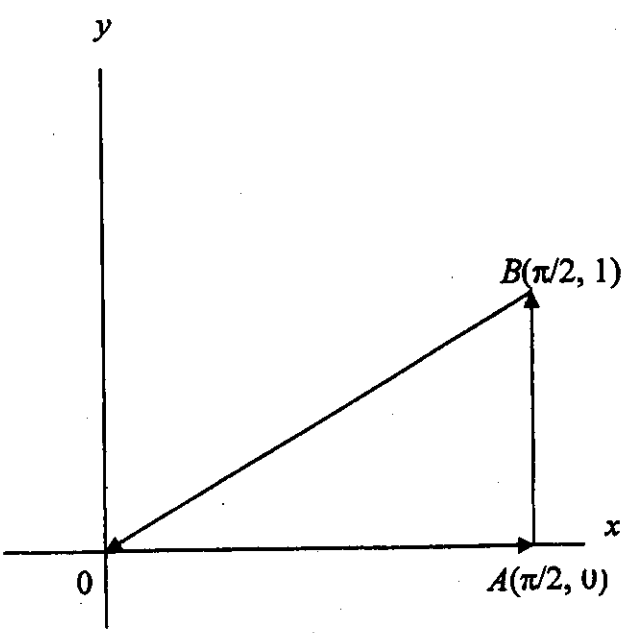

Fig. 1