※ 考生請注意：本試題 ☑可 ☐不可 使用計算機

Problem 1. (25%) Given a multi-input multi-output system as

$$\begin{pmatrix} Y_1(s) \\ Y_2(s) \end{pmatrix} = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \begin{pmatrix} U_1(s) \\ U_2(s) \end{pmatrix}$$

Where $g_{11}(s) = \frac{2s}{s+1}$, $g_{12}(s) = \frac{1}{s+2}$, $g_{21}(s) = \frac{1}{s+3}$, $g_{22}(s) = \frac{s}{s+4}$,

$Y_1(s)$ is the Laplace transform of $y_1(t)$. If $u_1(t)$ is a unit step function and $u_2(t)$ is an impulse function (a) Find $Y_1(s)$ and $Y_2(s)$ (10%). (b) Calculate $y_1(t)$ and $y_2(t)$, you can make proper assumptions if necessary (15%).

Problem 2. (25%) Given a plant with a transfer function of G(s), the output $c(t) = 1 - 2e^{-t}u(t) + e^{-2t}\cos t$ when the input $r(t) = e^{-t}u(t)$. The initial conditions are zero and u(t) is the unit-step function. (a) Find the transfer function G(s) (10%) (b) Draw root locus of the unit feedback system for the plant G(s) and explain how to use the root-locus for the design of a proportional feedback controller (15%).

Problem 3. (25%)

The steering of a car can be represented by the block diagram (shown in Fig. 3).

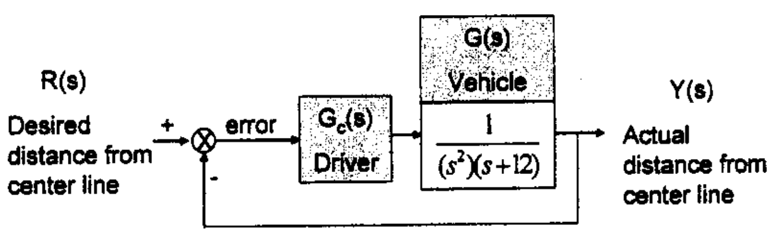

Figure 3: Block diagram of problem 3

(a) (5%) Suppose that the action of the driver on the steering wheel is proportional to the error between the actual and the desired path of the car, i.e., $G_c(s)=K_p$ where K_p is the proportional gain, sketch the root locus as K_p varies from zero to infinity and determine if the closed-loop system is stable when $K_p = 2$.

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(b) (20%) When a more experienced driver takes over, the driver's action can be represented by a PD controller, i.e., $G_c(s)=K_p(s+a)$.

(b1) (10%) Find the value of "a" that will result in a dominant response with a damping ratio 0.707 and a natural frequency of 2 (rad/s).

(b2) (5%) Sketch the root locus of the system as K_p varies from zero to infinity and determine the range of K_p for the closed-loop system to be stable (clearly show the intersection of the asymptotes with the real axis).

(b3) (5%) Use the root locus plot in (b2) to discuss the effect of increasing "a" on the stability of the system.

Problem 4. (25%)

Given a plant $G(s)$ and its closed-loop block diagram with controller $G_c(s)$ shown in Fig. 4,

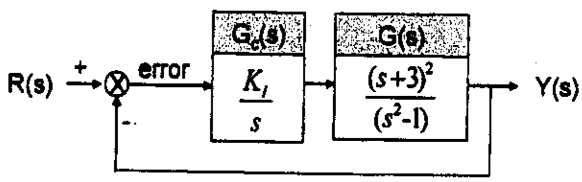

Figure 4: Block diagram of problem 4.

(a) (10%) Draw the Nyquist diagram and use Nyquist stability criterion to determine the range of K_I such that the closed-loop system is stable.

(b) (10%) Assume that you design a controller with $K_I = 2$, please draw the Bode diagram of the open loop $Gc(s)G(s)$ in the frequency interval 0.1~100 (rad/s).

(c) (5%) Continue (b); now, you are asked to implement your controller with hardware which will result in a pure time delay "T". Please find the range of T such that the closed-loop system is still stable. *(note: the gain cross over frequency in (b) is 3.3 (rad/s))*