系所組別　機械工程學系甲、乙、丙、丁、戊組
考試科目　工程數學　　　　　　　　　　　　　　　　　　　　考試日期　0307　節次　3

※ 考生請注意　本試題 □可　☑不可 使用計算機

1. Find the general solution of the following differential equation.

$$x^3 \frac{d^3y}{dx^3} + x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = x^{-2} \qquad (25\%)$$

2. (A) Show that the differential equation

$$x^4 y'' + \lambda y = 0 \qquad \text{(a)}$$

has an irregular singular point at $x = 0$ (3%)

(B) Show that the substitution $t = \frac{1}{x}$ into equation (a) yields the differential

equation

$$\frac{d^2y}{dt^2} + \frac{2}{t} \frac{dy}{dt} + \lambda y = 0 \qquad \text{(b)} \qquad (5\%)$$

which now has a regular singular point at $t = 0$

(C) Find two series solutions of the equation in part (b) about the singular point
$t = 0$ (14%)

(D) Express each series solution of the original equation in terms of elementary
functions. (3%)

3. By applying Green theorem to calculate $\oint_C \vec{F}(\vec{r}) \ d\vec{r}$ for a domain R whose

boundary is the closed curve C , where $\vec{F}(\vec{r}) = gradient\,(\sin x \cdot \cos y)$, $\vec{r}$ is the

position vector and C : $25x^2 + 9y^2 = 225$ (25 %)

4. (i) Find the Laurent series of $f(z) = \dfrac{1}{(z^2 - 3z + 2)(z^2 + 4)}$ with the

center at $z = 1$ (10%)

(ii) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{1}{(x^2 - 3x + 2)(x^2 + 4)} dx$ (15%)